Inferring Conduct to Guide Strategic Trade Policy

Eileen Ce Bian[*] Keith Head[†] Scott Orr [‡]

February 5, 2025

Abstract

In oligopoly market structures, optimal trade policies depend on the conduct of competing firms. Whether firms compete with prices or quantities as the strategic variables has a major quantitative and sometimes qualitative impact on the design of optimal trade policies. Conduct is not directly observable, but we develop an econometric method to infer it from data on prices, quantities, and a cost shifter such as tariffs. We apply the method to the widely used case of a constant elasticity of substitution demand and *ad valorem* tariffs. Using simulations, we show how policymakers could infer conduct from estimation and thereby generate domestic welfare gains from strategic trade policy when conduct is *ex-ante* unknown. Several caveats from the literature remain important, but our method mitigates concerns over imperfect information on conduct.

1 Introduction

In the early 1980s, trade economists investigated whether a country has a unilateral incentive to subsidize its exports or put tariffs on imports in oligopolistic industries (Brander 1995). The modeling strategy, whose canonical statement can be found in Brander and Spencer (1985) involves a two-stage game. A government chooses a trade policy in the first stage taking into account the way this policy will affect competition in the second stage. The core idea is that the government's ability to manipulate firms' incentives via subsidies will shift profits foreign firms to domestic firms. In the third-country market

[*]Sauder School of Business, University of British Columbia, ce.bian@sauder.ubc.ca.
[†]Sauder School of Business, University of British Columbia and CEPR, keith.head@sauder.ubc.ca.
[‡]Sauder School of Business, University of British Columbia, scott.orr@sauder.ubc.ca.

model where the home firm does not consume the product, raising domestic profits net of the subsidy is sufficient for a welfare increase. Eaton and Grossman (1986) pointed out that the benefits from export subsidies relied on the Cournot conduct assumption in Brander and Spencer (1985). With Bertrand competition, export *taxes* would be the optimal intervention.

Conduct matters for policy, but industries do not have labels stating whether the firms compete *à la* Bertrand or Cournot. Consider two examples. Airlines (an industry studied by Brander and Zhang 1990) appear to set prices since their websites post fares by route and time. However, the airlines also choose the flight frequency and plane size on each route. Thus, they also set quantities. Car makers post "manufacturer suggested retail prices" but they also allocate vehicles to each dealer and then offer incentives to make sure that dealers do not accumulate excessive inventories. These cases suggest that direct observation of firms' practices with respect to prices and quantities does not reveal unambiguously which abstraction fits best for a given industry. Since conduct needs to be known to determine which class of trade policy interventions are welfare improving, the difficulty of ascertaining conduct has led some authors to conclude that strategic trade policy is unlikely to be feasible in practice. (Grossman, 1986, p. 48) provides the definitive statement of this view: "...[W]e do not now (and may never) have sufficient knowledge and information to merit the implement of a policy of industrial targeting."

In this paper, we propose an approach to carrying out strategic policy in settings where the model of conduct is unknown, and show in Monte Carlo exercises that the approach may improve welfare by moderate amounts. In our setting, policy makers have access to price and quantity data for a series of export markets, but they do not know all the demand, cost, and conduct primitives needed to determine optimal policy. We show that standard tools from industrial organization (Berry 1994, Berry and Haile 2014) can be used to *estimate* the unknown demand and cost parameters, while a particular conduct model can then be tested for using a Hausman (1978) test. Using simulations for a simple CES differentiated product model, we show that our method can generate systematic welfare gains for a government that unilaterally chooses export subsidies (or taxes) to maximize domestic welfare.

We are far from the first to consider the problem of recovering unknown conduct. Indeed, a voluminous literature on this problem in the empirical industrial organization literature has developed two broad approaches to tackling this problem. First, conduct might be *estimated*, in the sense that there exists a (usually continuous) parameter that indexes different conduct models that can be recovered from price and quantity data.

Examples include the estimation of various "conjectural variation" models (e.g. Iwata 1974, Bresnahan 1981, 1982, Brander and Zhang 1990) or the more recent "rival profit weight" models developed in Ciliberto and Williams (2014) and Miller and Weinberg (2017). Alternatively, one can impose a conduct model and then formally *test* whether a particular conduct model fits the data (e.g. Bresnahan 1987,Gasmi et al. 1992, Backus et al. 2021, Duarte et al. 2023).

Corts (1999) raised concern over the internal consistency problem in the estimation approach using conjectural variations. Our paper fits within the second major approach to conduct, which emphasizes *testing*. The recent literature on testing conduct in industrial organization literature includes Backus et al. 2021 and Duarte et al. 2023. These papers test for model consistency by checking the difference between predicted markups from two alternative supply models *conditional* on demand estimates that are obtained without imposing a supply model. This testing procedure require that researchers ignore potentially valuable supply-side information when estimating demand-side parameters.

Our conduct testing method develops a different approach to assessing model consistency. It relies on the well known insight that the demand-side of an imperfectly competitive model can be consistently identified *without perfect knowledge of supply side conduct.* Imposing a conduct model and estimating both demand and cost parameters simultaneously, on the other hand, can potentially provide an estimator that is more efficient, but at the cost of generating an inconsistent estimator of the demand parameters when the imposed conduct model is incorrectly specified. This means that for each potential conduct regime R, we have two estimators; a demand based estimator that is always consistent (provided sufficient instruments exist and the demand system is well specified), and a demand and conduct based estimator that is only consistent if the conduct model is correct. This is precisely the sort of setting where the two-estimator specification test proposed Hausman (1978) can be used, and rejection of the null can be understood as a rejection of the conduct model R.

We provide a proof-of-concept of this testing approach using a series of Monte Carlo simulations which illustrate that implementing strategic trade policy when conduct is *ex-ante* unknown can still be welfare improving when guided by our proposed method. We consider a setting where a policy maker observes prices, quantities, and *ad valorem* tariffs over a series of differentiated product oligopoly export markets. To conduct strategic trade policy, the policy maker needs to estimate demand, the unobserved marginal costs of the competitors in each market, as well as the appropriate model of conduct. In our simulations, the policy maker estimates these objects and tests for Cournot and Bertrand conduct using our proposed Hausman test. We show that the Hausman test is able to discriminate between models even when there are a modest number of export markets, and that implementing strategic trade policy whenever it is possible to discriminate between conduct models leads to moderate welfare gains.

Our approach allows researchers to obtain more precise estimates of demand parameters by augmenting demand only moments with supply side moments, and then checking the distance between the demand only and the demand-and-supply estimates as a model consistency check. This is a different notion of model consistency than that proposed by Backus et al. (2021) who Duarte et al. (2023), who instead rely on GMM criterion functions defined over supply-side moments conditional on a supply-side agnostic estimate of the demand system.

We directly test the relevance of our metric of model consistency by examining the performance of a policy maker who, instead of running a formal Hausman test to determine which model fits the data, simply chooses the model (Bertrand or Cournot) that is *closest* to the demand only estimates. This particular decision rule has the desirable property that the policy maker always chooses *some* model to conduct trade policy, but has the added risk, relative to the Hausman test, that they may choose welfare *harming* trade policy when estimates are imprecise. In contrast, when we implement the Hausman test regime, the policy maker chooses *laissez-faire* if the test either accepts, or rejects, *both* models. This happens more often when there are few markets and therefore demand estimates are imprecise. Interestingly, we find that the "nearest neighbor" framework for choosing conduct and trade policy *outperforms* our Hausman test, in the sense that a social planner is able to obtain higher welfare gains by simply choosing the model of best fit, rather than only pursuing interventionist trade policy when we can definitively reject one model over the other.

The paper proceeds as follows. Section 2 returns to the Brander and Spencer (1985) setup, applying a constant elasticity of substitution (CES) demand structure. This yields some familiar results but also a surprising new feature: Cournot conduct does not necessarily imply strategic substitutes with CES demand. Section 3 extends the model to include multiple firms and multiple markets. Section 4 proposes a conduct-inference approach based on the Hausman (1978) test. We carry out a series of simulations in Section 5 to investigate the welfare effects of strategic trade policy based on our conduct-inference approaches. Section 6 concludes with some important caveats.

2 Two-firm third-market model

To provide the basic intuition for the importance of firm conduct to optimal trade policy under oligopoly, we begin by presenting classic results for the two-firm third-market model introduced by Brander and Spencer (1985). We develop these results for a CES demand system, which we will rely on in subsequent sections where we extend the environment to settings with more firms and markets.

2.1 Environment

Assume that there are two production origins and each has one firm. Both firms sell to a third market with market size M only. Denote the domestic firm's export price as p and the foreign firm's export price as p^*. The quantities of exports are x and y, and A and A^* are demand shifters.

$$x = M \frac{p^{-\sigma} A^{\sigma-1}}{p^{1-\sigma} A^{\sigma-1} + (p^*)^{1-\sigma} (A^*)^{\sigma-1}}$$

$$y = M \frac{(p^*)^{-\sigma} (A^*)^{\sigma-1}}{p^{1-\sigma} A^{\sigma-1} + (p^*)^{1-\sigma} (A^*)^{\sigma-1}}$$

The constant marginal costs of production are c and c^* respectively. Now the domestic country imposes an ad-valorem export subsidy or tax s on its firm to maximize its welfare $W(s)$. The profits of the domestic firm and the foreign firm are

$$\pi(x, y; s) = (1+s)px - cx$$

$$\pi^*(x, y; s) = p^* y - c^* y$$

where s is the ad-valorem subsidy provided to the domestic exporting firm. Note that if $s < 0$, the government taxes the domestic firm's exports, and rebates those taxes back to consumers.[1] Given s, both firms choose quantities or prices depending on the conduct to maximize their profits simultaneously. The welfare of the domestic country is given by

$$W(s) = \pi(x, y; s) - spx = px - cx,$$

[1]Implicit in this model is an "outside" perfectly competitive sector that produces a numeraire good that consumers purchase and consume with the domestic profits and taxes rebated to them. We set the price of this numeraire good to 1.

2.2 Strategic substitutes, complements, and optimal policy

Following Brander and Spencer (1985), Eaton and Grossman (1986), and Brander (1995), we introduce a two-stage game to solve for the optimal subsidy or tax given the conduct. In the first stage, the domestic country imposes export subsidy (or tax) s.[2] In the second stage, firms in each country simultaneously choose quantities sold to each market or prices (depending on conduct) to maximize their profits given s. In what follows, we refer to the *strategic variable* z as the choice variable (either price of quantity) for the appropriate Nash equilibrium in the second stage.

One of the core insights from the third market model is that taxes or subsidies can help commit the domestic firm to acting "as-if" they were the first-mover in a sequential game. This can allow the domestic firm to obtain higher profits—rebated back to domestic consumers—at the expense of the foreign firm; this is often called the "profit shifting" motive for strategic trade policy. However, the exact type of action— and therefore policy—a firm would want to use as a first mover depends on whether the choice variables are strategic complements—where an increase in strategy variable z by the domestic firm induces the foreign firm to increase their z^*—or strategic substitutes, where an increase in z by domestic firms induces the foreign firm to decrease z^*. In general, a strategic variable will be a strategic substitute in this class of game whenever $\pi_{z^*z} \equiv \frac{\partial^2 \pi}{\partial z \partial z^*} < 0$, while the strategic variable will be a strategic complement when $\pi_{z^*z} > 0$ (Bulow et al. 1985).

In the homogeneous good Cournot game considered in Brander and Spencer (1985), quantities are always strategic substitutes. This means the policy maker will want to encourage the domestic firm commit to a higher level of output, as this will induce the foreign firm to decrease their output, allowing the domestic firm to capture more profit; this is achieved by subsidizing the domestic firm's output. On the other hand, in the differentiated product setting considered in Eaton and Grossman (1986), prices are strategic complements. This means that the domestic firm can potentially extract more profit by softening competition and committing to higher prices, which the foreign firm will then "accommodate" by raising their price as well; this commitment to higher prices can be imposed by the government choosing an export tax, rather than a subsidy.

The same intuition approximately applies to the differentiated product, CES case developed here, although with an important ambiguity for the Cournot case which we shall become clearer in a moment. It is straightforward to verify that the optimal export sub-

[2]If $s > 0$ there is a subsidy, while if $s < 0$ there is a tax.

sidy or tax s^C of the domestic country under Cournot competition will satisfy:[3]

$$s^C = -\pi_y \frac{\pi^*_{yx}}{p\pi^*_{yy} + xp_x\pi^*_{yy} - xp_y\pi^*_{yx}} \tag{1}$$

while under Bertrand competition, it will satisfy

$$s^B = -\frac{\pi_{p^*}\pi^*_{p^*p}}{px_p\pi^*_{p^*p^*} + x\pi^*_{p^*p^*} - px_{p^*}\pi^*_{p^*p}} \tag{2}$$

We derive these results formally in Appendix A and Appendix B, with the above results nesting the general case of many firms and countries to the special case of two export countries with one exporting firm each. Since $\pi_y < 0$, $\pi_{p^*} > 0$, $p\pi^*_{yy} + xp_x\pi^*_{yy} - xp_y\pi^*_{yx} < 0$ and $px_p\pi^*_{p^*p^*} + x\pi^*_{p^*p^*} - px_{p^*}\pi^*_{p^*p} > 0$ (see Appendix B) it is easy to verify that whenever $\pi^*_{yx} < 0$ or $\pi^*_{p^*p} < 0$ (strategic substitutes) the optimal policy is a subsidy with $s^C > 0$ or $s^B > 0$, while whenever $\pi^*_{yx} > 0$ or $\pi^*_{p^*p} > 0$ (strategic complements), the optimal policy is a tax.

To determine the sign of the optimal policy in the CES demand model considered here, it is useful to define S_x as the revenue-based market share for good x, and S_y as the revenue based market share for good y.[4] We can solve for π^*_{yx} or $\pi^*_{p^*p}$ as a function of exogenous and endogenous variables as follows:

$$\begin{aligned} &\textbf{Cournot:} \quad \pi^*_{yx} = \frac{\partial^2\pi^*}{\partial x \partial y} = p^*_x + yp^*_{yx} = -\left(\frac{\sigma-1}{\sigma}\right)^2 \frac{1}{M} pp^*\left(1 - 2S_y\right) \\ &\textbf{Bertrand:} \quad \pi^*_{p^*p} = \frac{\partial^2\pi^*}{\partial p \partial p^*} = y_p + p^* y_{p^*p} - c^* p_{p^*p} = (1-\sigma)^2 M \frac{p^* - c^*}{p^*} S_y^2 S_x \frac{1}{p^* p} > 0 \end{aligned} \tag{3}$$

Since CES demand is a special case of the model developed in Eaton and Grossman (1986), we obtain the standard result that Bertrand pricing always involves strategic substitutes, and therefore the optimal trade policy involves an export tax. On the other hand, the Brander and Spencer (1985) intuition for strategic substitutes with Cournot—which was developed for a homogeneous goods model—does not always apply to the CES differentiated goods context considered here. In fact, under CES demand, Cournot competition will only involve strategic substitutes on the foreign firm's reaction function whenever the foreign firm is small, i.e. $S_y < \frac{1}{2}$; conversely, whenever the foreign firm is large, i.e. $S_y > \frac{1}{2}$ the Cournot model involves strategic complements. This can lead

[3]Here, a_b denotes the partial derivative of a with respect to b.

[4]Thus, we have $S_x = \frac{xp}{xp+yp^*}$ and $S_y = \frac{yp^*}{xp+yp^*}$.

to non-monotonicities in the firm's reaction functions under Cournot, which can make determining the optimal policy complex.

Figure 1 visualizes the best response functions of the two firms under two different modes of conduct; Cournot in panels (a) and (b), and Bertrand in panels (c) and (d). Panels (a) and (b) show that the best response functions are not monotonic under Cournot competition. Starting from the symmetric equilibrium and giving an export subsidy $s_1 > 0$ to the domestic firm will increase its export quantities and make its market share exceed 0.5. Under this scenario, $\pi^*_{yx} < 0$ and the best response of the foreign firm is to decrease its export quantities (strategic substitutes). Similarly, when imposing an export tax on the domestic firm, in the new equilibrium, $\pi^*_{yx} > 0$, both firms will decrease their export quantities (strategic complements).

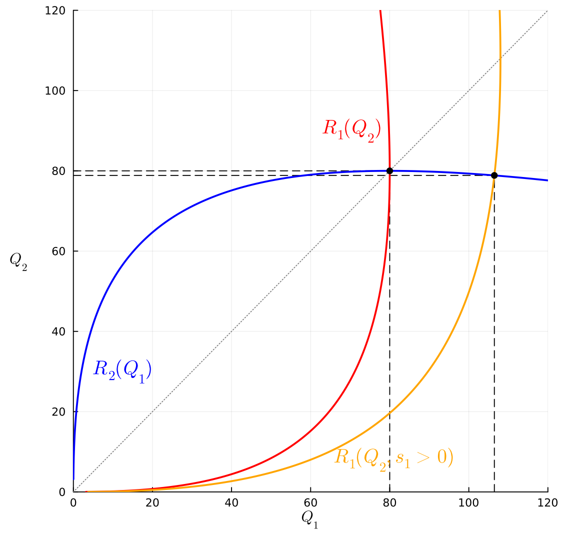
Figure 1c and Figure 1d show that the best response functions are monotonically increasing under Bertrand competition, given the two firms are always strategic complements. Therefore, the prices of the two firms will move in the same direction with export subsidy or tax imposed on the domestic firm.

The slopes of the reaction functions are crucial for understanding optimal policy. We will see that under Bertrand competition, a robust result is that the optimal policy is an export tax. With Cournot competition, export subsidies are often preferable, but this depends on market sizes as we shall see in the next section.

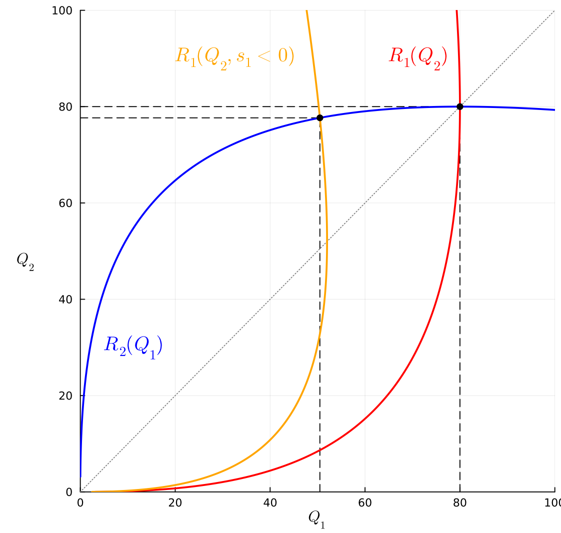
2.3 Numerical examples in the base case

In general, correctly implementing the socially optimal strategic trade policy depends on whether firms compete in prices or quantities, i.e. the model of conduct. As we move toward our final econometric exercise where we consider an environment where a policy maker wishes to conduct strategic trade policy while not knowing the appropriate conduct model, it is useful to first consider what this simple model tells us about the costs and benefits of getting the conduct model right. In particular, we have already shown that the *sign* of the optimal policy may flip if the policy maker gets the model of conduct wrong. But what about magnitudes? In this subsection, we show that under some parameterizations of our model, the benefit of being right may be small, while the costs of being wrong can be sizeable. This suggests that having to *estimate* conduct is a potentially risky enterprise, where econometric uncertainty is important take seriously.

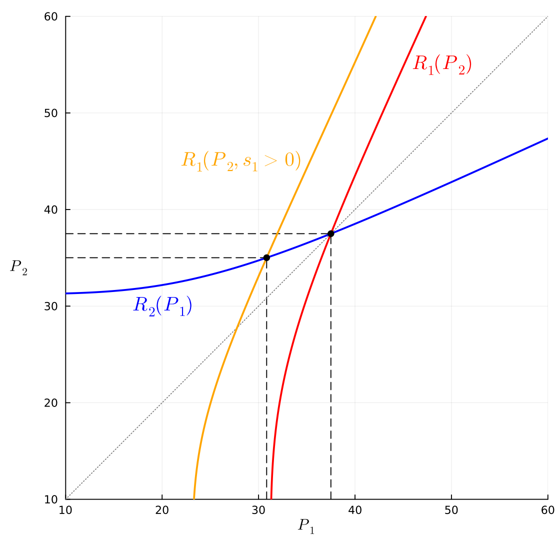
Figure 1: Best response functions with export subsidy or tax under CES demand



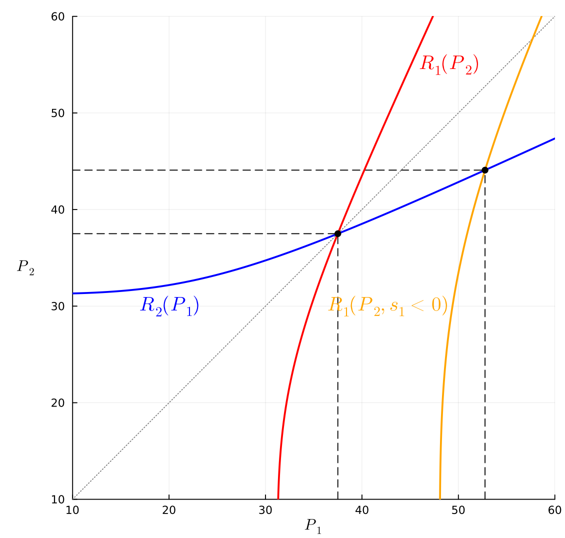
(a) Cournot, export subsidy



(b) Cournot, export tax



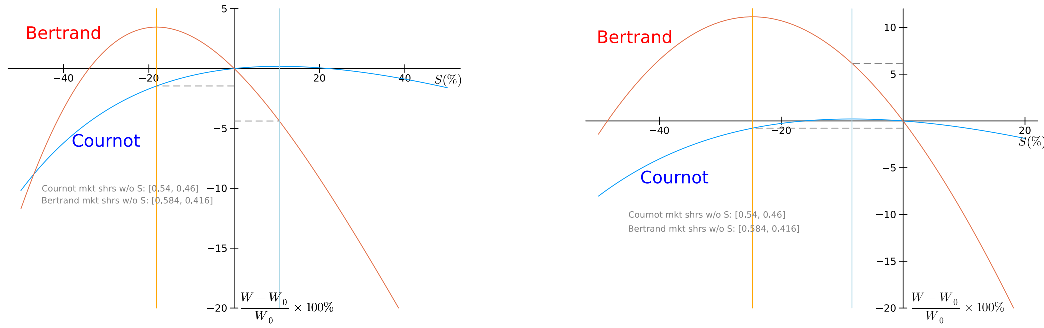
(c) Bertrand, export subsidy



(d) Bertrand, export tax

Note: The red and blue lines are best-response functions of firm 1 and firm 2, respectively, with CES demand with $A = A^*$ and $c = c^* = 0.25$. The orange line is the best response function of firm 1 after imposing an *ad valorem* export subsidy $s_1 = 0.35$ (panels a and c) or tax $s_1 = -0.35$ (panels b and d).

Figure 2: Welfare change with export subsidy/tax in two-firm third-market model



(a) Country with the larger firm imposes export subsidy/tax

(b) Country with the smaller firm imposes export tax

Note: The x-axis is the *ad valorem* subsidy on the exporter from country 1. The y-axis is the percentage welfare change relative to no subsidy. The vertical orange and blue lines show the welfare-maximizing subsidies for Bertrand and Cournot conduct. The rival country sets $s_2 = 0$. We set $\sigma = 5$, $A = \exp(0.6)^{0.25}$, $A^* = \exp(1.4)^{0.25}$, $c = c^* = 1 + \exp(1.0)$.

Figure 2 visualizes the welfare change of a country under different export subsidy or tax in terms of marginal cost for a given set of exogenous parameters. Assume that the domestic firm is the one that has a larger market share in the third market. In this example, as shown in Figure 2a the domestic country will impose an optimal export subsidy $s^C > 0$ under Cournot competition and impose an optimal tax $s^B < 0$ under Bertrand competition. Note that the welfare loss of inferring the wrong conduct and imposing the subsequent s^C or s^B is substantial. If the true conduct is Cournot and the domestic country mistakenly assumes Bertrand, the welfare loss is around 1.5% with the export tax. The loss is even larger if the true conduct is Bertrand; under this specific DGP, if the domestic country imposes an export subsidy based on Cournot, welfare falls by 4.4%. As the foreign firm has a smaller market share at initial equilibrium, as shown in Figure 2b, the foreign country will impose an optimal export tax $s^C < 0$ under Cournot competition and also impose an optimal tax $s^B < 0$ under Bertrand competition. In this case, the optimal export tax based on the Cournot competition will still improve the welfare of the foreign country if the true conduct is Bertrand, but the welfare gain is smaller than the optimal export tax based on the true conduct. On the other hand, there is welfare loss if the foreign country imposes an export subsidy based on the Bertrand competition while the true conduct is Cournot.

While these numbers come from a particular parametric example, note that under alternative parameter settings, the welfare loss of inferring the wrong conduct could be larger or smaller. Below, we run a series of simulations—where we draw demand and supply parameters for a collection of independent export markets—to quantify the potential magnitude of these gains and loses on average.

3 Extending the third-market model

Having now worked through the canonical two firm third market model, we now extend our CES environment to a setting with many firms, countries, as well as a domestic sector in each market. This extension is important to accommodate econometric estimation in real world settings.

3.1 Environment

Assume that there are C production origins and each has N_c exporting firms. The set of exporting firms in country c is denoted by $\mathcal{I}_c$. There are also M sales markets without any exporters. The total number of markets is $C + M$.

Each market m has a local firm indexed by $i = 0$ that only sells to market m; i.e., they do not export.[5] Each firm only produces a single variety. For simplicity, we assume that each exporting firm sell in all markets including its domestic market, i.e., the total number of products in market m is $1 + \sum_{c=1}^{C} N_c$. The quantity sold by firm i from country c in market m is parameterized as

$$q_{icm} = Y_m \frac{p_{icm}^{-\sigma} A_{icm}^{\sigma-1}}{\left(\frac{p_{0mm}}{A_{0mm}}\right)^{1-\sigma} + \sum_{c'} \sum_{i' \in \Omega_{c'm}} \left(\frac{p_{i'c'm}}{A_{i'c'm}}\right)^{1-\sigma}} \tag{4}$$

where Y_m is the total demand in market m, p_{icm} is the price, A_{icm} is a firm-market specific demand shifter, and Ω_{cm} is a set of firms from production origin c operating in market m. For simplicity, we assume that every exporting firm sells in all markets.

The inverse demand function is then given by:

$$p_{icm}(\mathbf{q}_m) = Y_m \frac{q_{icm}^{-\frac{1}{\sigma}} A_{icm}^{\frac{\sigma-1}{\sigma}}}{(q_{0mm} A_{0mm})^{\frac{\sigma-1}{\sigma}} + \sum_{c'} \sum_{i' \in \Omega_{c'm}} (q_{i'c'm} A_{i'c'm})^{\frac{\sigma-1}{\sigma}}}$$

[5] The presence of the local firm implies that aggregates sales of the foreign oligopolists is not fixed.

The variable profit of firm i from country c in market m without any tax or subsidy is

$$\pi_{icm} = p_{icm}q_{icm} - c_{icm}q_{icm}$$

The total profit of firm i from country c is

$$\pi_{ic} = \sum_{m=1}^{C+M} \pi_{icm}$$

Aggregating profits of all firms and subtracting the total subsidy paid to (or adding the total tax collected from) all exporting firms in country c, the total welfare of country c is

$$W_c(\mathbf{s}) = \sum_{i \in \mathcal{I}_c} \pi_{ic} - \sum_{m \neq c} s_{cm} \sum_{i \in \mathcal{I}_c} p_{icm}q_{icm} \tag{5}$$

where s_{cm} is the *ad valorem* export subsidy (or tax) imposed by country c on all firms that export to market m.

To make estimation and simulation of this model feasible, it is useful to parameterize the demand and supply sides of the model. Specifically, the quantity sold by firm i from country c in market m is parameterized as

$$q_{icm} = Y_m \frac{p_{icm}^{-\sigma} \exp(\xi_{icm})}{p_{0mm}^{1-\sigma} \exp(\xi_{0mm}) + \sum_{c'} \sum_{i' \in \Omega_{c'm}} p_{i'c'm}^{1-\sigma} \exp(\xi_{i'c'm})} \tag{6}$$

where $\xi_{icm} \equiv (\sigma - 1) \ln (A_{icm})$

The constant marginal cost of selling from production origin c to market m is parameterized as

$$c_{icm} = \tau_{c(i)m} \exp(\omega_{icm})$$

where $\tau_{c(i)m}$ denotes *ad valorem* tariffs charged on goods moving from origin country c to market m, and ω_{icm} is a firm-origin-destination specific (log) productivity shock.

Firms choose prices to maximize profits in each market, which leads to the standard markup rule,

$$p_{icm} = \frac{\epsilon_{icm}^R}{\epsilon_{icm}^R - 1} \tau_{c(i)m} \exp(\omega_{icm}).$$

where ϵ_{icm}^R is a firm's perceived demand elasticity, whose particular functional form varies with the assumed conduct model $R = B$ (Bertrand) or $R = C$ (Cournot). If we assume that firms choose price and quantity pair taking their rivals prices as fixed as in Bertrand competition, then one can show that

$$\epsilon_{icm}^{B} = \sigma + (1-\sigma)S_{icm}\left(\mathbf{p}_m, \boldsymbol{\xi}_m\right), \tag{7}$$

where $S_{icm}\left(\mathbf{p}_m, \boldsymbol{\xi}_m\right) \equiv \frac{p_{icm}q_{icm}}{p_{0mm}q_{0mm}+\sum_{c'}\sum_{j\in\Omega_{c'm}} p_{icm}q_{icm}} = \frac{p_{icm}^{1-\sigma}\exp(\xi_{icm})}{p_{0mm}^{1-\sigma}\exp(\xi_{0mm})+\sum_{c'}\sum_{j\in\Omega_{c'm}} p_{jc'm}^{1-\sigma}\exp(\xi_{jc'm})}$ is the revenue market share of firm i from production origin c in market m. These market shares are determined by the vector of prices chosen by each firm operating in market m ($\mathbf{p}_m$), as well as the vector of demand shocks for each firm operating in market m ($\boldsymbol{\xi}_m$).

On the other hand, if firms choose their individual price and quantity pairs taking their rival's *outputs* as given, the relevant demand elasticities will then be given by:

$$\epsilon_{icm}^{C} = \frac{\sigma}{1+(\sigma-1)S_{icm}\left(\mathbf{p}_m, \boldsymbol{\xi}_m\right)}. \tag{8}$$

The difference in perceived demand elasticities under Bertrand and Cournot then leads to a separate *pricing rule* for each conduct model $R = B, C$:

$$p_{icm} = \begin{cases} \left(\frac{1}{(\sigma-1)(1-S_{icm})}+1\right)\tau_{c(i)m}\exp(\omega_{icm}) = \mu^{B}(\sigma, S_{icm})\tau_{c(i)m}\exp(\omega_{icm}) & \text{if } R = B \\ \frac{\sigma}{(\sigma-1)(1-S_{icm})}\tau_{c(i)m}\exp(\omega_{icm}) = \mu^{C}(\sigma, S_{icm})\tau_{c(i)m}\exp(\omega_{icm}) & \text{if } R = C \end{cases} \tag{9}$$

where $\mu^{R}(\sigma, S_{icm})$ denotes the markup function under pricing rule $R = B, C$.[6]

Note that these equilibrium pricing rules generate weakly higher markups under Cournot competition compared to Bertrand. This is because when firms believe their rivals quantities are fixed when they choose their price and quantity pair, individual price cuts must implicitly be matched by their rivals for their quantities to remain fixed. This implicitly softens price competition relative to a model where rival prices are assumed to be fixed in response to price cuts, thereby leading firms to exercise a higher degree of market power under Cournot competition.

3.2 Optimal export subsidy or tax

As with the two-firm third-market model, we introduce a two-stage game to solve the optimal export subsidy /tax, s_{cm}^{C} and s_{cm}^{B}. Now in the first stage, country c imposes a set of market-specific per-unit export subsidies (or taxes) s_{cm} on all exporting firms. Since production involves constant marginal costs, each firm's profit maximization problem is

[6] The own elasticity derivations are provided for Cournot and Bertrand in appendix A.

independent in each market. In Appendix A we derive the following formulas which characterize the optimal subsidy, s_{cm}^C and s_{cm}^B, under Cournot and Bertrand competition:

$$s_{cm}^C = \frac{\sum_{i\in\mathcal{I}_c} \frac{\partial \pi_{icm}(\mathbf{Q})}{\partial q_{0mm}} \frac{\mathrm{d}q_{0mm}}{\mathrm{d}s_{cm}} + \sum_{i\in\mathcal{I}_c} \sum_{j\neq i} \frac{\partial \pi_{icm}(\mathbf{Q})}{\partial q_{jcm}} \frac{\mathrm{d}q_{jcm}}{\mathrm{d}s_{cm}} + \sum_{i\in\mathcal{I}_c} \sum_{k\neq c} \sum_{j\in\mathcal{I}_k} \frac{\partial \pi_{icm}(\mathbf{Q})}{\partial q_{jkm}} \frac{\mathrm{d}q_{jkm}}{\mathrm{d}s_{cm}}}{\sum_{i\in\mathcal{I}_c} \left(p_{icm} \frac{\mathrm{d}q_{icm}}{\mathrm{d}s_{cm}} + q_{icm} \sum_k \sum_{j\in\mathcal{I}_k} \left(\frac{\partial p_{icm}}{\partial q_{jkm}} \frac{\mathrm{d}q_{jkm}}{\mathrm{d}s_{cm}} + \frac{\partial p_{icm}}{\partial q_{0mm}} \frac{\mathrm{d}q_{0mm}}{\mathrm{d}s_{cm}} \right) \right)}$$

$$s_{cm}^B = \frac{\sum_{i\in\mathcal{I}_c} \frac{\partial \pi_{icm}(\mathbf{P})}{\partial p_{0mm}} \frac{\mathrm{d}p_{0mm}}{\mathrm{d}s_{cm}} + \sum_{i\in\mathcal{I}_c} \sum_{j\neq c} \frac{\partial \pi_{icm}(\mathbf{P})}{\partial p_{jcm}} \frac{\mathrm{d}p_{jcm}}{\mathrm{d}s_{cm}} + \sum_{i\in\mathcal{I}_c} \sum_{k\neq c} \sum_{j\in\mathcal{I}_k} \frac{\partial \pi_{icm}(\mathbf{P})}{\partial p_{jkm}} \frac{\mathrm{d}p_{jkm}}{\mathrm{d}s_{cm}}}{\sum_{i\in\mathcal{I}_c} \left(q_{icm} \frac{\mathrm{d}p_{icm}}{\mathrm{d}s_{cm}} + p_{icm} \sum_k \sum_{j\in\mathcal{I}_k} \left(\frac{\partial q_{icm}}{\partial p_{jkm}} \frac{\mathrm{d}p_{jkm}}{\mathrm{d}s_{cm}} + \frac{\partial q_{icm}}{\partial p_{0mm}} \frac{\mathrm{d}p_{0mm}}{\mathrm{d}s_{cm}} \right) \right)} \quad (10)$$

The optimality conditions in equation (10) show that the optimal tax or subsidy now depends on three sets of terms in the numerator. Moving from right to left, we have the effects of the subsidy on the firms from the country imposing the policy, the effects on firms from the rival countries and finally the effect on the output of the local firm in the target market. While we cannot make further progress on these equations analytically—as the optimal subsidy appears on both the left- and right- hand side of these expressions—we use them to solve for optimal subsidy and taxes numerically via fixed point iteration. Specifically, for each market m and given c, we solve for the subsidy or tax with starting value $s_{cm}^0 = 0$ based on equation (10), and then update the subsidy or tax in the right-hand side of the equation until convergence.

3.3 Numerical examples in the extended case

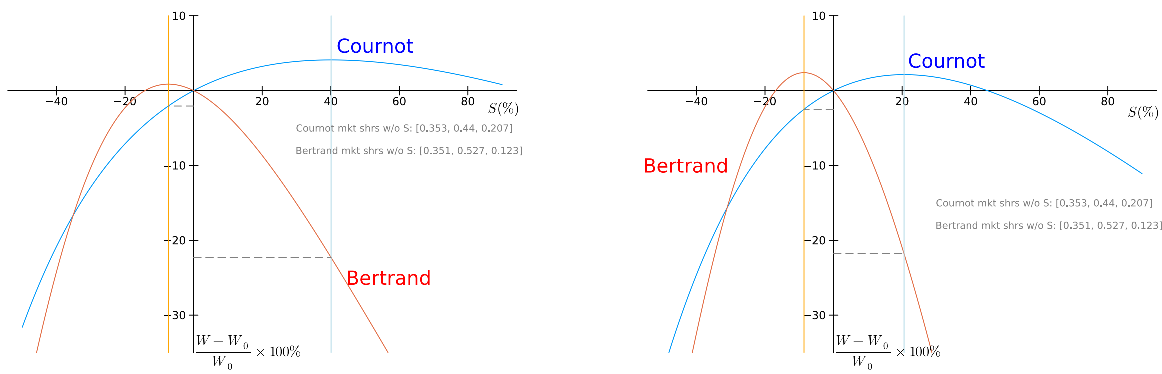
Now we modify the simulations in Section 2.3 to include an additional local firm in the third market. The data generating process is described in Appendix D. With three firms competing in the third market, the sign of the optimal export subsidy or tax becomes more ambiguous. Figure 3 shows a case where the local firm has a market share of less than a quarter of the market. In this case, both the exporters—both the country with a smaller market share, and the one with the larger share—will both want to impose an optimal export subsidy $s^C > 0$ under Cournot competition. Thus, it resembles the classic Brander and Spencer (1985) case despite the use of product differentiation and the presence of the domestic competitor. One novel point we see here is that the country with the larger market share wants to impose a bigger export subsidy. This runs somewhat counter to the idea of using a subsidy to overcome a weakness, but it makes sense here because bigger domestic firm will tend to face a rival with more negative slope of their reaction function.

Under Bertrand competition, we obtain the Eaton and Grossman (1986) result of an optimal tax ($s^B < 0$). In both cases there is notable welfare loss when the wrong conduct is inferred. The situation is particularly bad when the policy maker believes there is Cournot competition, but the true conduct is Bertrand. Then we see export subsidizing countries have greater than 20% losses in welfare. Inferring Bertrand is a less costly mistake.

Alternatively, Figure 4 shows a case where the local firm has a market share of more than 85% in the third market. Under this DGP, both countries impose an optimal export tax $s^B < 0$ under Bertrand and $s^C < 0$ under Cournot conduct. Interestingly, there is still a welfare gain even when the wrong conduct is inferred. However, inferring Cournot incorrectly leads the home country to impose a smaller than optimal export tax.

Two additional visualizations on welfare changes with multiple production origins and multiple firms are provided in Appendix F. The main takeaway from these examples is that knowing the correct conduct is always useful for designing optimal trade policy. Furthermore, when the two rival exporters dominate the third market (as in the classic models of the 1980s), then knowing conduct can avoid a sign error in optimal policy.

Figure 3: C = 2, N = 1, with local firm (market share of local firm $< 25\%$)

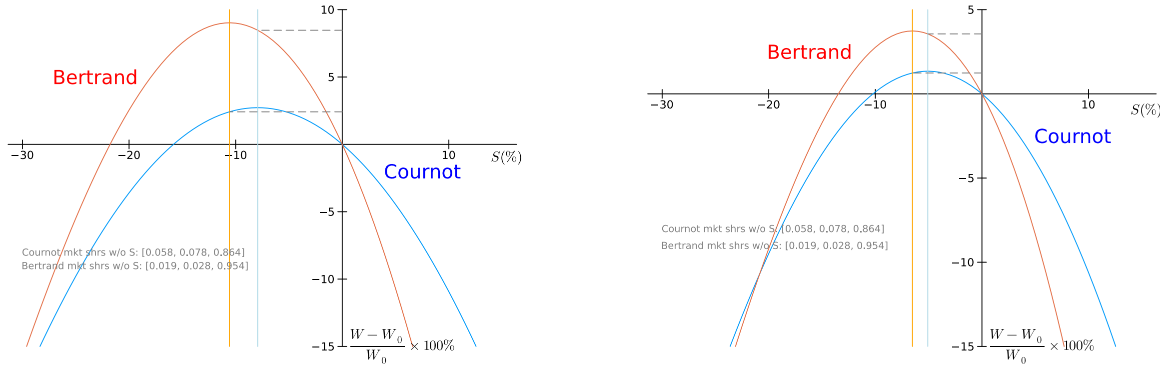


(a) Larger exporting country imposes export subsidy/tax

(b) Smaller exporting country imposes export subsidy/tax

Note: The x-axis is the *ad valorem* subsidy on the exporter from country 1. The y-axis is the percentage welfare change relative to no subsidy. The vertical orange and blue lines show the welfare-maximizing subsidies for Bertrand and Cournot conduct. The rival country sets $s_2 = 0$. Denoting the two production origins as countries 1 and country 2, and the "third-market" as country 3, we set $\sigma = 5$, $\xi_{113} = 0.6$, $\xi_{123} = 1.4$, $\xi_{033} = -6.0$, $\omega_{113} = \omega_{123} = \omega_{033} = 1.0$, $\tau_{13} = \tau_{23} = 1 + \exp(1.0)$, $\tau_{33} = 1.0$, $Y_3 = 1$. Equilibrium prices, quantities and optimal export subsidies or taxes are solved via fixed-point iteration.

Figure 4: C = 2, N = 1, with local firm (market share of local firm $> 85\%$)



(a) Larger exporting country imposes export tax

(b) Smaller exporting country imposes export tax

Note: The x-axis is the *ad valorem* subsidy on the exporter from country 1. The y-axis is the percentage welfare change relative to no subsidy. The vertical orange and blue lines show the welfare-maximizing subsidies for Bertrand and Cournot conduct. The rival country sets $s_2 = 0$. Denoting the two production origins as countries 1 and country 2, and the "third-market" as country 3, we set $\sigma = 5$, $\xi_{113} = 0.8$, $\xi_{123} = 1.2$, $\xi_{033} = 6.0$, $\omega_{113} = \omega_{123} = \omega_{033} = 1.0$, $\tau_{13} = \tau_{23} = 1 + \exp(1.0)$, $\tau_{33} = 1.0$, $Y_3 = 1$.

4 Inferring conduct through a Hausman Test

In the previous sections, we presented variants of the classic Brander and Spencer (1985) third country model of strategic trade policy. In practice, realizing the potential welfare gains from optimal policy requires that policy makers know (i) the demand and costs of the competing firms, and (ii) the appropriate model of conduct. All of these objects are potentially unobserved by the policy maker. Importantly, since the *sign* of optimal policy may not even be known if conduct is unknown, some researchers have concluded that strategic trade policy is unlikely to provide a successful guide to implementing welfare improving trade policies.

In this section, we develop a potential path forward for realizing the gains from strategic trade policy; *estimating* demand and costs and then *testing* for a particular model of conduct. The setting is one in which policy makers have access of data on the prices and quantities sold for various firms in a series of third markets m, thereby allowing the policy maker to estimate demand and cost using standard tools from the industrial organization literature (e.g. Berry 1994, Berry and Haile 2014). We show that the techniques used to recover demand and costs from price and quantity data naturally suggest a straightforward to implement *Hausman test* for the appropriate model of conduct.

We present this test in the context particular parameter model developed in Section 3; this is both for expositional convenience, as well as to elucidate the DGP we will use in the Monte Carlo is Section 5, where we consider the *performance* of this approach in a setting where the policy maker needs to estimate demand, cost, and conduct. However, we also show in Appendix C that this test can be generalized to more flexible models of demand and cost.

In what follows, we assume that the policy maker knows the CES functional form in Equation (4), but does not know $(\sigma, \xi_m, \omega_m)$, nor the mode of conduct R; rather, they only observe for a series of markets m then prices, quantities, and *ad valorem* tariffs $(\mathbf{p}_m, \mathbf{q}_m, \tau_m)$. To tackle this problem, we first show that this model readily allows the policy maker to recover a consistent estimate of demand and productivity shocks, (ξ_m, ω_m), as long as they are able to generate a consistent estimate of σ. The fact that unknown demand and supply parameters are *invertible* conditional on the global demand parameter σ is key to developing our conduct test; specifically we rely on the commonly noted insight that as long as supply-side instruments exist, demand parameters can be estimated *without* imposing a conduct model. On the other hand, recovering supply side parameters generally requires knowledge of demand parameters *and* imposing a conduct a model; this provides a series of overidentifying restrictions that will be more efficient when a researcher imposes the correct conduct model, but inconsistent when the wrong conduct model is imposed. This naturally leads to a Hausman test for conduct.

4.1 Invertibility of (ξ_m, ω_m) given σ

While the policy maker does not directly know anything about each firm's costs, if they know how each firm chooses their prices, they can recover estimates of marginal costs by relying on the fact that profit maximizing firms will choose prices to equate marginal revenues with marginal costs. Specifically, if the policy maker is able to construct an estimate of marginal revenue for each firm, then they can recover marginal cost estimates as in Rosse (1970) by simply setting this unknown object equal to (estimated) marginal revenue. For the model developed here, this mapping is implicit in the pricing rule (9); rearranging this expression yields the following equation which can be used to recover estimates of the productivity shocks form all firm products, $\widehat{\omega_{icm}}$, given observed data and and estimate of the elasticity of substitution $\widehat{\sigma}$:

$$\widehat{\omega}_{icm}(\widehat{\sigma}, R) = \ln p_{icm} - \ln\left(\mu^R(\widehat{\sigma}, S_{icm})\tau_{c(i)m}\right) \tag{11}$$

A similar inversion exists for the unknown demand shocks as well; in particular, it is straightforward to show that (4) implies:

$$\xi_{icm} - \xi_{0mm} = \ln \frac{S_{icm}}{S_{0cm}} - (1-\sigma)\ln\left(\frac{p_{icm}}{p_{0mm}}\right)$$

The above provides a mapping between relative prices, market shares, the elasticity and substitution, and *relative demand shocks*, $\xi_{icm} - \xi_{0cm}$. Since CES demand systems are homogeneous of degree zero in demand shocks, the overall scale of these demand shocks are not relevant for counterfactuals. As a result, for the purpose of determining the optimal subsidy, a policy maker can normalize ξ_{0mm} to zero, recovering the remaining demand shocks as:

$$\widehat{\xi}_{icm}(\widehat{\sigma}) = \ln \frac{S_{icm}}{S_{0cm}} - (1-\widehat{\sigma})\ln\left(\frac{p_{icm}}{p_{0mm}}\right) \tag{12}$$

Together, Equation (11) and Equation (12) provide a way for the policy maker to recover all the relevant unknown demand and supply shocks for purpose of determining optimal policy, given σ and the mode of conduct R. In the next two subsections, we then show how the policy maker can use these properties to generate estimating equations for σ, which then provide over-identifying restrictions to test for the appropriate conduct model R.

4.2 Estimating σ

To recover the global demand parameter, σ, a policy maker can rely on standard demand estimation techniques. A simple tool for this purpose that is available when demand is CES is based on the following transformation of demand, which generates a linear estimating equation (Berry 1994, Björnerstedt and Verboven 2016):

$$\ln \frac{S_{icm}}{S_{0cm}} = (1-\sigma)\ln\left(\frac{p_{icm}}{p_{0mm}}\right) + \xi_{icm} - \xi_{0mm} \tag{13}$$

Here, the structural error term—which we call a firm's relative demand shock—is given by $\xi_{icm} - \xi_{0mm}$. While $(1-\sigma)$ could be estimated by applying OLS to Equation (13), in general this procedure will not generate consistent estimates of σ as equilibrium prices p_{icm} will tend to be correlated with relative demand shocks for two reasons. First, according to both pricing rules in Equation (9), firms with higher revenue shares will tend to charge higher prices. Since higher revenue shares can directly be generated by a firm experiencing a larger relative demand shock, this generates a positive correlation between prices and revenue shares. Second, since prices also move with marginal costs, prices will be correlated with relative demand shocks whenever there is a systematic correlation between demand shocks and productivity. For example, higher quality goods may be more difficult to produce, which will tend to generate a positive correlation between A_{icm} and ω_{icm} (Hottman et al. 2016, Jaumandreu and Yin 2018, Orr 2022, Forlani et al. 2023). Alternatively, more productive firms may simply select into producing higher quality goods, as emphasized by Kugler and Verhoogen (2012), which would generate a negative correlation between ξ_{icm} and ω_{icm}.

Consistent estimation of Equation (13) generally requires an *instrumental variable* that is excluded from the demand system, correlated with price, and uncorrelated with relative demand shocks. Equation (9) provides a natural candidate for such an instrument; tariffs $\tau_{c(i)m}$. Notably, consumers do not directly care about tariffs (except through their impact on the price they pay)—thereby excluding this variable from direct consideration in the demand system—while tariffs will generally increase prices through the equilibrium pricing rules. In what follows, we assume tariffs are not set in an endogenous way—or more formally, that $\mathbb{E}(\xi_{icm} - \xi_{0cm})\,\tau_{c(i)m} = 0$— so that Equation (13) can be estimated using a standard linear IV estimator, with $\tau_{c(i)m}$ serving as the instrument. This provides a researcher with an estimated value of σ, which we will denote by $\widehat{\sigma}^D$.

Note that when justifying the use of $\tau_{c(i)m}$ as an instrument, we could rely on either the Bertrand or Cournot pricing rule; in fact, as long as firms internalize costs when choosing prices, exogenous cost shifters such as tariffs can be used as instruments to identify demand. This means that our identification strategy for $\widehat{\sigma}^D$ is *robust* to the policy maker's lack of knowledge on the exact mode of conduct.

On the other hand, note that if the policy maker *knew* the mode of conduct, they could obtain more efficient estimates of σ by leveraging their knowledge of the firm's exact pricing rule in Equation (9). More precisely, the pricing rule for a given conduct model R provides a series of overidentifying restrictions that, when correctly specified, should lead to more efficient estimates of σ. This is because a correctly specified pricing rule tells use how market shares should vary across markets conditional on σ,which provides a series of further moments to help pin down σ. We generate an estimator that takes advantage of this idea by formulating another estimating equation for σ by substituting Equation (9) into Equation (13), yielding:

$$\ln \frac{S_{icm}}{S_{0cm}} = (1-\sigma)\left(\ln \mu^R(\sigma, S_{icm}) - \ln \mu^R(\sigma, S_{0cm})\right) + (1-\sigma)\ln \tau_{c(i)m}$$
$$= + \underbrace{(1-\sigma)\omega_{icm} + \xi_{icm} - \xi_{0mm}}_{\equiv \nu_{icm}} \tag{14}$$

Treating the sum of the productivity and relative demand shocks as the new structural residual, we can obtain a new estimate of σ by estimating the above model by nonlinear GMM. Since market shares enter the markup rule for each proposed model of conduct $R = \{B, C\}$, we still need to rely on an instrumental variables formulation of the estimator—rather than a nonlinear least squares procedure — as S_{icm} will tend to be correlated with both relative demand shocks and productivity shocks. For this purpose, we implement a nonlinear GMM estimator that relies on the following moment condition $\mathbb{E}\nu_{icm}\tau_{c(i)m} = 0$. Validity of this moment condition requires that tariffs be uncorrelated with both demand and productivity shocks; a sufficient condition for which is that tariffs be completely exogenous.[7]

Estimating Equation (14) for a given conduct model R only provides a single moment for identifying σ. In practice, this variation alone may be more or less efficient at identifying σ than estimating Equation (13) alone. To generate an estimator for σ that will tend to be more efficient under a correctly specified model of conduct R, we propose estimating both Equation (13) and Equation (14) simultaneously using a system GMM estimator. In practice, we implement this through a two step procedure which chooses an approximation to the optimal weighting matrix for overidentified models as in Hansen (1982). We denote the estimate of σ obtained by this estimator as $\widehat{\sigma}^R$,where R denotes the particular model of conduct imposed in Equation (14).

While $\widehat{\sigma}^R$ will tend to be more efficient than $\widehat{\sigma}^D$ when R is correctly chosen, $\widehat{\sigma}^R$ will be inconsistent if the wrong conduct model is imposed on the data. This is because the pricing rule estimator imposes a series of mispecified moments to identify σ. In what follows, we take advantage of the fact that these two estimators generate a tradeoff between consistency and efficiency, which is precisely what a Hausman test was designed to exploit in model testing.

[7] An important property we use here is that the policy maker knows that the passthrough rate of tariffs to marginal costs is 1; this means the above equation only has one unknown parameter to be estimated (σ),rather than σ and the passthrough rate of tariffs to marginal costs. As a result, we only need a single instrument to identify this model. While this is appropriate for a setting where *ad-valorem* tariffs are available and exogenous, we show in Appendix C that we can also generalize our approach to settings where the researcher only has access to a series of supply-side instruments where the pass-through rate of the instrument to marginal costs is unknown and therefore must also be estimated.

4.3 Hausman Test

The Hausman test, developed in Hausman (1978), is a standard econometric tool to test for whether two estimators of the same object are converging to the same probability limit. This type of test is particularly useful when it is known that one of the estimators will only be consistent when a stronger set of conditions hold, while another estimator is consistent under weaker conditions. For example, the Hausman test is often used in a panel data context to test for whether random effects (which imposes the strong condition that individual specific effects are uncorrelated with other regressors) are appropriate, with the alternative, less efficient, estimator being the fixed effects estimator, which does not require uncorrelated of the individual fixed effects.

In our context, the Hausman test can be computed as follows:

$$H = \frac{\left(\widehat{\sigma}^D - \widehat{\sigma}^R\right)^2}{\widehat{\text{Var}}(\widehat{\sigma}^D - \widehat{\sigma}^R)}$$

where $\widehat{\text{Var}}(\widehat{\sigma}^D - \widehat{\sigma}^R)$ is the estimated variance of $\widehat{\sigma}^D - \widehat{\sigma}^R$.[8] Note that the Hausman test simply tests for whether the two estimators are close enough to one another, adjusting for the variance of each estimator.

Hausman (1978) shows that this test statistic is asymptotically distributed as a chi-square distribution with one degree of freedom. As a result, we can then use this statstic to formally test the hypothesis that $\widehat{\sigma}^D = \widehat{\sigma}^R$. Rejection of this hypothesis leads to rejection of a particular conduct model R; this is because we know the demand esitmator is $\widehat{\sigma}^D$ is consistent for any conduct model. As a result, differences in $\widehat{\sigma}^D$ and $\widehat{\sigma}^R$ tells us the conduct model R is likely mispecified. On the other hand, failing to reject means that a particular conduct model R is potentially capable of explaining the data.

In the next section of this paper, we examine the behavior of this testing procedure in a series of Monte Carlo exercises. Specifically, we consider a policy maker who observes prices, quantity, and tariff data for a series of markets m, and wishes to conduct strategic trade policy in each of these markets to improve overall domestic welfare. We assume they follow the above estimation procedures for σ, and then implement the above Hausman test based on significance levels of 0.01, 0.05, and 0.1 for rejection of the null hypothesis. Note that for a policy maker to conduct this policy exercise, they need to decide

[8]While this object can be difficult to compute, we follow Cameron and Trivedi (2005) and compute it using the bootstrap. Since our model involves a series of independent export markets, we randomly sample export markets in our implementation, which implicitly clusters standard errors by market.

on how to resolve situations where either both conduct models R are either rejected or not rejected. Since strategic trade policy can lead to fairly large welfare loses when a policy maker sets subsidies according to the wrong conduct model, we assume they conservatively choose *laissez faire* (no tax or subsidy) whenever both conduct models are rejected or not rejected according to this procedure.

5 Simulating strategic trade policy with unknown conduct

In this section, we carry out a series of simulations to investigate the effectiveness of the policy-maker's strategic trade policy based on our conduct-inference approach in the extended third-market model. We assume that only one country c imposes a set of export subsidies or taxes s_{cm} on its exporting firms. The first conduct-inference approach we adopt is based on the Hausman test described in Section 4. Specifically, if the Hausman test rejects the null hypothesis that the conduct is Cournot and fails to reject the null hypothesis that the conduct is Bertrand, we infer that the conduct is Bertrand. If the Hausman test rejects the null hypothesis that the conduct is Bertrand and fails to reject the null hypothesis that the conduct is Cournot, we infer that the conduct is Cournot. If the Hausman test fails to reject both null hypotheses or rejects both null hypotheses, we cannot infer conduct.

The second approach is simply based on selecting the model whose estimate $\widehat{\sigma}^R$ is closest to the demand IV estimate $\widehat{\sigma}^D$; we call this the "nearest neighbor" approach. This approach focuses attention our somewhat novel metric of model consistency, which treats models with smaller values of $\left(\widehat{\sigma}^D - \widehat{\sigma}^R\right)^2$ as fitting the data "better." Note, however, that this approach runs the risk of potentially generating high welfare *losses* than the Hausman test approach. In particular, the Hausman test may reject, or fail to reject, both models in some settings due to uncertainty in the parameter estimates; in this case we assume the policy maker does nothing, so consumers are no better or worse off. The nearest neighbor approach instead always selects at least one of either Cournot or Bertrand, which runs a greater risk of choosing the wrong policy than the Hausman approach. Since the nearest neighbor policy is more interventionist than the Hausman test, we expect it it will tend to generate more *variable* welfare gains than the more conservative Hausman testing procedure; this naturally suggests that a policy maker's risk aversion should likely determine which policy regime is preferable in practice.

The simulations we focus on here are based on the extended third-market model with

$C = 2$ production origins and $N = 1$ exporting firm in each production origin.[9] The data generating process is the same as Section 3.3 described in Appendix D and Table 1 summarizes the parameters chosen in the simulations. We assume that production origin 1 is the country that imposes export subsidies or taxes on its exporting firm.

Table 1: Parameters chosen in the simulations, C = 2, N = 1

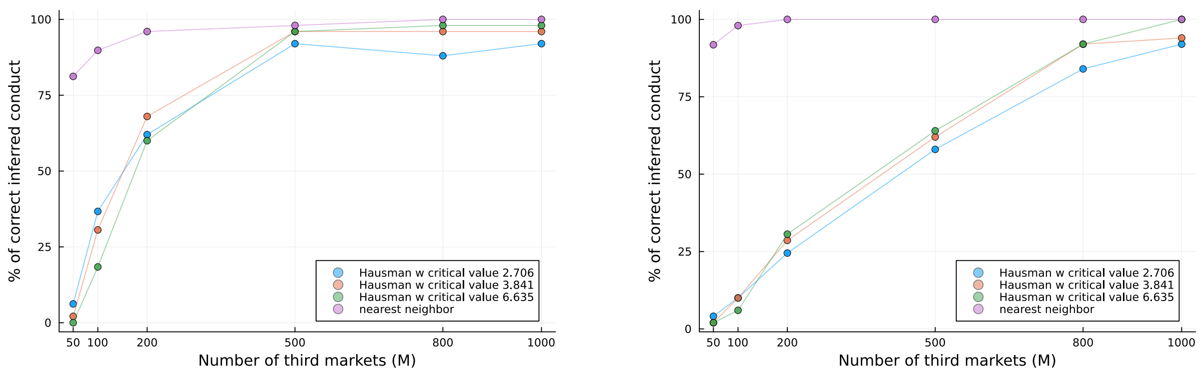
variable	**value**	**mean**	**variance**	**definition**
Y_m		1.0	1.0	Market size
ξ_{icm}		0.0	30.0	Demand shifter of importing goods
ξ_{0mm}		-3.0	1.0	Demand shifter of local goods
ω_{icm}		0.0	0.5	Cost shifter of exporters
ω_{0mm}		0.0	0.5	Cost shifter of local firms
ψ_{cm}		1.0	2.0	Trade cost shifter
σ	4.0			Elasticity
$\rho(\xi_{icm}, \omega_{icm})$	0.5			Demand-cost correlation of importing goods
$\rho(\xi_{0mm}, \omega_{0mm})$	0.5			Demand-cost correlation of local goods

To evaluate the performance of the two conduct-inference approaches, we first check the percentage of simulations where we correctly infer conduct. The results are shown in Figure 5, where we plot the percent of correctly predicted markets using Hausman tests with 1%, 5%, and 10% significance levels, as well as the nearest-neighbor approach. On the x-axis, we vary the number of third markets to get a sense of how many independent markets (effectively, our sample size) a policy maker must observe to correctly identify the right conduct model. We find that conduct is easier to identify under the Bertrand DGP, compared to the Cournot DGP. While we identify the correct mode of conduct a bit under 100 % of the time when we consider 1000 independent markets for all three Hausman tests, the identification of Cournot conduct appears to be somewhat more sensitive to sample size than Bertrand, with the % of times we correctly identify conduct immediately beginning to decline as we decrease the sample size. Since nearest neighbor still gets 100% of the simulations right here—i.e., our point estimates are still consistent with the right model—this tells us this is purely a precision issue, leading our social planner to behave conservatively (choose *laissez faire*) when there is not sufficient evidence for them to choose one model of the other. On the other hand, the proportion of simulations where we can correctly identify Bertrand conduct under a Hausman test remains relatively constant

[9] Results for three rather than two countries are reported in in Appendix E. Qualitatively results are similar, although the magnitude of the gains are smaller. It also becomes somewhat harder to identify conduct under Bertrand competition when we observe more firms.

until we start considering less than 500 export markets. After decreasing the number of export markets even further—less than 200—nearest neighbor starts to occasionally rule in favour of the *wrong* model, which is precisely when the conservativeness of a testing procedure that accounts for uncertainty is likely to be most valuable.

Figure 5: Percent of correct inferred conduct, $C = 2$, $N = 1$



(a) Bertrand DGP

(b) Cournot DGP

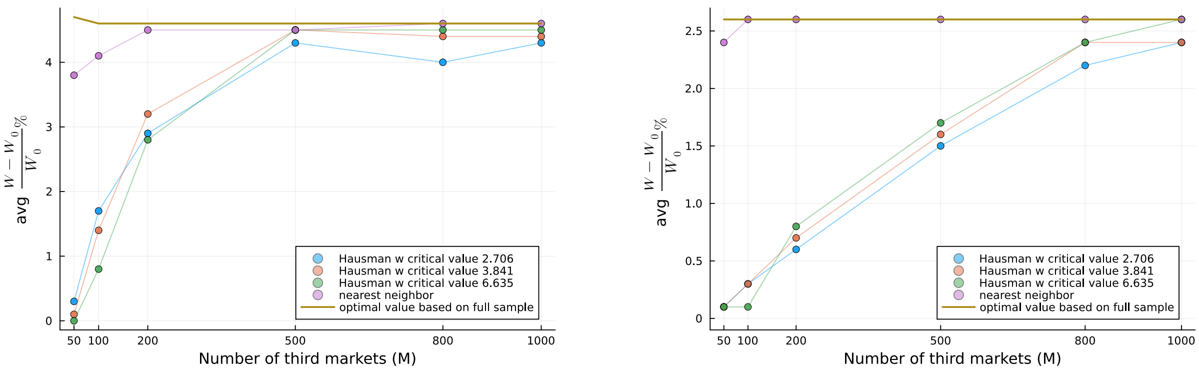
Note: The y-axis is the percentage of correct conduct inferences. The x-axis is the number of third markets.Each line represents an inference approaches. The Hausman tests are conducted with 1%, 5%, and 10% significance levels. The nearest neighbor approach selects the model whose estimate $\widehat{\sigma}^R$ is closest to the demand IV estimate $\widehat{\sigma}^D$.

In the next set of results, we turn to welfare properties of choosing strategic trade policy after inferring conduct. We assume that the policy-maker observes all equilibrium prices, quantities, market shares and bilateral trade costs $\tau_{c(i)m}$ to estimate σ using both demand IV and GMM approaches. If the policy-maker cannot infer conduct, the optimal export subsidy or tax is set to zero. If the policy-maker can infer conduct, the optimal export policy is solved for using the first-order condition of the social welfare function in Equation (10) via fixed-point iterations. To compute the welfare change, we first solve for the new equilibrium with estimated $\widehat{\sigma}^R$, $\widehat{\boldsymbol{\xi}}$ and $\widehat{\boldsymbol{\omega}}$ given the inferred conduct R. Specifically, we refer to the methodogy described in Section 4.1 to obtain estimated $\widehat{\xi}$ and $\widehat{\omega}$ given $\widehat{\sigma}^R$ and observed data. Given $\widehat{\sigma}^R$, $\widehat{\boldsymbol{\xi}}$, $\widehat{\boldsymbol{\omega}}$ and constant market size Y_m, the new equilibrium prices and quantities can be solved via fixed-point iteration, and the welfare at new equilibrium is computed based on Equation (5).

Figure 6 shows the average welfare change under different conduct models and different numbers of observed third-markets. The solid brown lines represent the average welfare change under the true conduct and the optimal export subsidy or tax that is computed based on the true σ and simulated ξ and ω. The other lines represent the average welfare change under the inferred conduct and the optimal export subsidy or tax that is computed based on the estimated $\widehat{\sigma}^R$ and estimated data. Given the high success rate of nearest neighbor at identifying the correct mode of conduct documented in Figure 5, we unsurprisingly find that the welfare gains implied by nearest neighbor in Figure 6 are close to what an omniscient social planner would obtain. The same is true for the statistical Hausman tests when we observe many markets. However, these gains fall off quickly as we decrease the sample size under the Cournot DGP. On the other hand, under Bertrand, a social planner is able to generate average welfare gains close to 4% as long as the number of third markets is above 500. Interestingly, even in cases where conduct is hard to identify, i.e. both DGPs with less than 100 markets, the Hausman test still generates positive welfare gains.

The simulations suggest that our proposed metric of model misspecification—the difference between the demand only estimator and a demand-and-supply estimator—is very good at identifying the correct model of conduct, even when the policy maker has access to very little data. Simply choosing the model with the smallest value of $\left(\widehat{\sigma}^D - \widehat{\sigma}^R\right)^2$ as a model selection statistic appears to even outperform a formal Hausman test. That said, forcing the social planner to always pick one model—as we do under the nearest neighbor approach—does increase the risk of choosing the wrong model when estimates are imprecise, which will tend to generate more variable welfare benefits than the more conservative Hausman test. We illustrate this in Figure 7, which plots the standard deviation of welfare changes across simulations. We see that the nearest neighbor approach always generates a higher standard deviation in welfare than the Hausman test approach. As a result, while the Hausman test tends to generate lower gains on average, there is also less variability in the realized gains, which suggests that risk averse policy makers may prefer the more conservative testing procedure in some cases; i.e. when the number of markets is small.

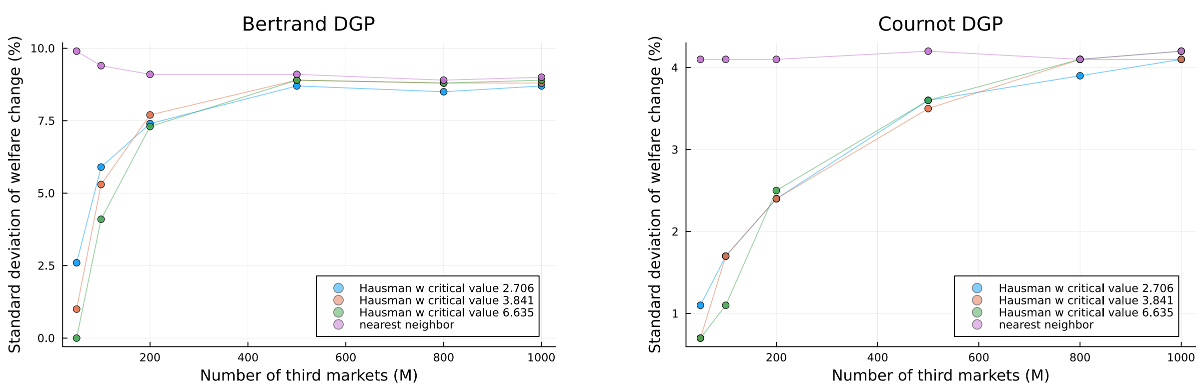
Figure 6: Average welfare change $\frac{W-W_0}{W_0}\%$, $C = 2$, $N = 1$



(a) Bertrand DGP (b) Cournot DGP

Note: The y-axis is the expected welfare change (in %). It is calculated as the average of $\frac{W-W_0}{W_0}\%$ over all cases in all simulations. If the policy-maker cannot infer the conduct, the optimal export subsidy or tax is set to zero, and the welfare change is zero. The x-axis is the number of third-markets. Each line represents a different approach to conduct inference. The Hausman tests are conducted with 1%, 5%, and 10% significance levels. The nearest neighbor approach selects the model whose estimate $\widehat{\sigma}^R$ is closest to the demand IV estimate $\widehat{\sigma}^D$.

Figure 7: Standard deviation of welfare change $\frac{W-W_0}{W_0}\%$



(a) Bertrand DGP (b) Cournot DGP

Note: The y-axis is the standard deviation of the welfare change (in %). The x-axis is the number of third-markets. If the policy-maker cannot infer the conduct, the optimal export subsidy or tax is set to zero, and the welfare change is zero. Each line represents a different approach to conduct inference. The Hausman tests are conducted with 1%, 5%, and 10% significance levels. The nearest neighbor approach selects the model whose estimate $\widehat{\sigma}^R$ is closest to the demand IV estimate $\widehat{\sigma}^D$.

6 Conclusion

In the 1980s concern over the dependence of optimal strategic trade policies on unobservable conduct led to skepticism about whether governments would have the information necessary to implement the optimal policies. That is even if we set aside concerns over retaliation and general equilibrium effects, it was natural to worry that a government that mistook conduct as Cournot when in fact it was Bertrand would implement the exact opposite policy of what would be optimal.

Our results here should not be seen as an argument for increased use of strategic trade policy. There remain many valid concerns with such policies. However, our results suggest that the natural response to not knowing conduct is to estimate it. We develop a method to do that using a fairly standard toolkit from econometrics. Our Monte Carlo results (under ideal circumstances) suggest that implementing our method could give rise to moderate welfare gains. A useful path for future work would be to implement the method on real world industry data.

References

Backus, M., Conlon, C., and Sinkinson, M. (2021). Common ownership and competition in the ready-to-eat cereal industry. Working paper, National Bureau of Economic Research.

Berry, S. T. (1994). Estimating discrete-choice models of product differentiation. *The RAND Journal of Economics*, pages 242–262.

Berry, S. T. and Haile, P. A. (2014). Identification in differentiated products markets using market level data. *Econometrica*, 82(5):1749–1797.

Björnerstedt, J. and Verboven, F. (2016). Does merger simulation work? Evidence from the Swedish analgesics market. *American Economic Journal: Applied Economics*, 8(3):125–164.

Brander, J. A. (1995). Strategic trade policy. *Handbook of international economics*, 3:1395–1455.

Brander, J. A. and Spencer, B. J. (1985). Export subsidies and international market share rivalry. *Journal of international Economics*, 18(1-2):83–100.

Brander, J. A. and Zhang, A. (1990). Market conduct in the airline industry: an empirical investigation. *The RAND Journal of Economics*, pages 567–583.

Bresnahan, T. F. (1981). Departures from marginal-cost pricing in the American automobile industry: Estimates for 1977–1978. *Journal of Econometrics*, 17(2):201–227.

Bresnahan, T. F. (1982). The oligopoly solution concept is identified. *Economics letters*, 10(1-2):87–92.

Bresnahan, T. F. (1987). Competition and collusion in the American automobile industry: The 1955 price war. *The Journal of Industrial Economics*, pages 457–482.

Bulow, J. I., Geanakoplos, J. D., and Klemperer, P. D. (1985). Multimarket oligopoly: Strategic substitutes and complements. *Journal of Political economy*, 93(3):488–511.

Cameron, A. C. and Trivedi, P. K. (2005). *Microeconometrics: methods and applications*. Cambridge university press.

Ciliberto, F. and Williams, J. W. (2014). Does multimarket contact facilitate tacit collusion? inference on conduct parameters in the airline industry. *The RAND journal of economics*, 45(4):764–791.

Corts, K. S. (1999). Conduct parameters and the measurement of market power. *Journal of Econometrics*, 88(2):227–250.

Duarte, M., Magnolfi, L., Sølvsten, M., and Sullivan, C. (2023). Testing firm conduct. *Quantitative Economics, Forthcoming*.

Eaton, J. and Grossman, G. M. (1986). Optimal trade and industrial policy under oligopoly. *The Quarterly Journal of Economics*, 101(2):383–406.

Forlani, E., Martin, R., Mion, G., and Muûls, M. (2023). Unraveling firms: Demand, productivity and markups heterogeneity. *The Economic Journal*, 133(654):2251–2302.

Gasmi, F., Laffont, J. J., and Vuong, Q. (1992). Econometric analysis of collusive behavior in a soft-drink market. *Journal of Economics & Management Strategy*, 1(2):277–311.

Grossman, G. M. (1986). Strategic export promotion: a critique. In Krugman, P. R., editor, *Strategic trade policy and the new international economics*, pages 47–68. MIT Press Cambridge, MA.

Hansen, L. P. (1982). Large sample properties of generalized method of moments estimators. *Econometrica: Journal of the econometric society*, pages 1029–1054.

Hausman, J. A. (1978). Specification tests in econometrics. *Econometrica*, 46(6):1251–1271.

Hottman, C. J., Redding, S. J., and Weinstein, D. E. (2016). Quantifying the sources of firm heterogeneity. *The Quarterly Journal of Economics*, 131(3):1291–1364.

Iwata, G. (1974). Measurement of conjectural variations in oligopoly. *Econometrica: Journal of the Econometric Society*, pages 947–966.

Jaumandreu, J. and Yin, H. (2018). Cost and product advantages: A firm-level model for the chinese exports and industry growth. *Working Paper*.

Kugler, M. and Verhoogen, E. (2012). Prices, plant size, and product quality. *The Review of Economic Studies*, 79(1):307–339.

Miller, N. H. and Weinberg, M. C. (2017). Understanding the price effects of the miller-coors joint venture. *Econometrica*, 85(6):1763–1791.

Orr, S. (2022). Within-firm productivity dispersion: Estimates and implications. *Journal of Political Economy*, 130(11):2771–2828.

Rosse, J. N. (1970). Estimating cost function parameters without using cost data: Illustrated methodology. *Econometrica: Journal of the Econometric Society*, pages 256–275.

A Derivation of optimal export subsidy/tax in extended third-market model

A.1 Cournot

The revenue of firm i from country c in market m is

$$R_{icm}(\mathbf{Q}) = p_{icm}(\mathbf{Q})q_{icm} = Y_m \frac{\left(q_{icm}A_{icm}\right)^{\frac{\sigma-1}{\sigma}}}{\left(q_{0mm}A_{0mm}\right)^{\frac{\sigma-1}{\sigma}} + \sum_{c'}\sum_{i'}\left(q_{i'c'm}A_{i'c'm}\right)^{\frac{\sigma-1}{\sigma}}}$$

The profit of firm i from country c with export subsidy or tax is

$$\begin{aligned}
\pi_{ic}(\mathbf{Q}) &= \sum_m \pi_{icm}(\mathbf{Q}) = \sum_m \left[R_{icm}(\mathbf{Q}) - c_{icm}q_{icm}\right] + \sum_{m\neq c} s_{cm}q_{icm} \\
&= \sum_m \left[Y_m \frac{\left(q_{icm}A_{icm}\right)^{\frac{\sigma-1}{\sigma}}}{\left(q_{0mm}A_{0mm}\right)^{\frac{\sigma-1}{\sigma}} + \sum_{c'}\sum_{i'}\left(q_{i'c'm}A_{i'c'm}\right)^{\frac{\sigma-1}{\sigma}}} - c_{icm}q_{icm}\right] \\
&+ \sum_{m\neq c} \left[s_{cm}Y_m \frac{\left(q_{icm}A_{icm}\right)^{\frac{\sigma-1}{\sigma}}}{\left(q_{0mm}\xi_{0mm}\right)^{\frac{\sigma-1}{\sigma}} + \sum_{c'}\sum_{i'}\left(q_{i'c'm}A_{i'c'm}\right)^{\frac{\sigma-1}{\sigma}}}\right]
\end{aligned}$$

The profit of firm j from another country k is

$$\begin{aligned}
\pi_{jk}(\mathbf{Q}) &= \sum_m \pi_{jkm}(\mathbf{Q}) = \sum_m \left[R_{jkm}(\mathbf{Q}) - c_{jkm}q_{jkm}\right] \\
&= \sum_m \left[Y_m \frac{\left(q_{jkm}A_{jkm}\right)^{\frac{\sigma-1}{\sigma}}}{\left(q_{0mm}A_{0mm}\right)^{\frac{\sigma-1}{\sigma}} + \sum_{c'}\sum_{i'}\left(q_{i'c'm}A_{i'c'm}\right)^{\frac{\sigma-1}{\sigma}}} - c_{jkm}q_{jkm}\right]
\end{aligned}$$

The welfare of country c is

$$W_c(s) = \sum_{i\in\mathcal{I}_c}\sum_m \pi_{icm}(\mathbf{Q}) - \sum_{i\in\mathcal{I}_c}\sum_{m\neq c} s_{cm}p_{icm}(\mathbf{Q})q_{icm}$$

The own price elasticity of demand is

$$\epsilon_{icm}^C \equiv -\left(\frac{\partial p_{icm}}{\partial q_{icm}}\frac{q_{icm}}{p_{icm}}\right)^{-1} = \left(\left(\frac{\frac{1}{\sigma}Y_m q_{icm}^{-\frac{1}{\sigma}-1}A_{icm}^{\frac{\sigma-1}{\sigma}}}{(q_{0mm}A_{0mm})^{\frac{\sigma-1}{\sigma}}+\sum_{c'}\sum_{i'\in\Omega_{c'm}}(q_{i'c'm}A_{i'c'm})^{\frac{\sigma-1}{\sigma}}}\right.\right.$$
$$\left.\left.+\frac{\frac{\sigma-1}{\sigma}Y_m q_{icm}^{-\frac{2}{\sigma}}A_{icm}^{\frac{2\sigma-2}{\sigma}}}{\left((q_{0mm}\xi_{0mm})^{\frac{\sigma-1}{\sigma}}+\sum_{c'}\sum_{i'\in\Omega_{c'm}}(q_{i'c'm}A_{i'c'm})^{\frac{\sigma-1}{\sigma}}\right)^2}\right)\frac{q_{icm}}{p_{icm}}\right)^{-1}$$
$$=\left(\frac{1}{\sigma}+\frac{\sigma-1}{\sigma}S_{icm}(\mathbf{p}_m,\boldsymbol{\xi}_m)\right)^{-1} = \frac{\sigma}{1+(\sigma-1)S_{icm}(\mathbf{p}_m,\boldsymbol{\xi}_m)}$$

A.1.1 Second stage (given s_{cm}):

First-order conditions

The first-order condition of firm i from country c in market $m \neq c$ is

$$\begin{aligned}\frac{\partial \pi_{icm}(\mathbf{Q})}{\partial q_{icm}} &= (1+s_{cm})q_{icm}\frac{\partial p_{icm}(\mathbf{Q})}{\partial q_{icm}} + (1+s_{cm})p_{icm}(\mathbf{Q}) - c_{icm} \\ &= (1+s_{cm})Y_m\frac{\sigma-1}{\sigma}\frac{(q_{icm}A_{icm})^{\frac{\sigma-1}{\sigma}}}{(q_{0mm}A_{0mm})^{\frac{\sigma-1}{\sigma}}+\sum_{c'}\sum_{i'}(q_{i'c'm}A_{i'c'm})^{\frac{\sigma-1}{\sigma}}}\frac{1}{q_{icm}} \\ &- (1+s_{cm})Y_m\frac{\sigma-1}{\sigma}\frac{(q_{icm}A_{icm})^{\frac{\sigma-1}{\sigma}}}{\left((q_{0mm}A_{0mm})^{\frac{\sigma-1}{\sigma}}+\sum_{c'}\sum_{i'}(q_{i'c'm}A_{i'c'm})^{\frac{\sigma-1}{\sigma}}\right)^2}(q_{icm}A_{icm})^{\frac{\sigma-1}{\sigma}}\frac{1}{q_{icm}} - c_{icm} \\ &= (1+s_{cm})Y_m\frac{\sigma-1}{\sigma}\frac{(q_{icm}A_{icm})^{\frac{\sigma-1}{\sigma}}}{(q_{0mm}A_{0mm})^{\frac{\sigma-1}{\sigma}}+\sum_{c'}\sum_{i'}(q_{i'c'm}A_{i'c'm})^{\frac{\sigma-1}{\sigma}}}\frac{1}{q_{icm}}\left(1-\frac{(q_{icm}A_{icm})^{\frac{\sigma-1}{\sigma}}}{(q_{0mm}A_{0mm})^{\frac{\sigma-1}{\sigma}}+\sum_{c'}\sum_{i'}(q_{i'c'm}A_{i'c'm})^{\frac{\sigma-1}{\sigma}}}\right) \\ &- c_{icm} \\ &= (1+s_{cm})\frac{\sigma-1}{\sigma}p_{icm}(1-S_{icm}) - c_{icm} = 0\end{aligned} \tag{15}$$

where S_{icm} is the market share of firm i from country c in market m. Similarly, the first-order condition of firm i from country c in market c is

$$\frac{\partial \pi_{icc}(\mathbf{Q})}{\partial q_{icc}} = q_{icc}\frac{\partial p_{icc}(\mathbf{Q})}{\partial q_{icc}} + p_{icc}(\mathbf{Q}) - c_{icc} = \frac{\sigma-1}{\sigma}p_{icc}(1-S_{icc}) - c_{icc} = 0 \tag{16}$$

The first-order condition of firm j from another country $k \neq c$ in any market m (including local firms in market m) is

$$\frac{\partial \pi_{jkm}(\mathbf{Q})}{\partial q_{jkm}} = q_{jkm}\frac{\partial p_{jkm}(\mathbf{Q})}{\partial q_{jkm}} + p_{jkm}(\mathbf{Q}) - c_{jkm} = \frac{\sigma-1}{\sigma}p_{jkm}(1-S_{jkm}) - c_{jkm} = 0 \tag{17}$$

Second-order conditions

$$\frac{\partial^2 \pi_{icm}(\mathbf{Q})}{\partial q_{icm}^2} = 2(1+s_{cm})\frac{\partial p_{icm}(\mathbf{Q})}{\partial q_{icm}} + (1+s_{cm})q_{icm}\frac{\partial^2 p_{icm}(\mathbf{Q})}{\partial q_{icm}^2} < 0 \tag{18}$$

$$\frac{\partial^2 \pi_{icc}(\mathbf{Q})}{\partial q_{icc}^2} = 2\frac{\partial p_{icc}(\mathbf{Q})}{\partial q_{icc}} + q_{icc}\frac{\partial^2 p_{icc}(\mathbf{Q})}{\partial q_{icc}^2} < 0 \tag{19}$$

$$\frac{\partial^2 \pi_{jkm}(\mathbf{Q})}{\partial q_{jkm}^2} = 2\frac{\partial p_{jkm}(\mathbf{Q})}{\partial q_{jkm}} + q_{jkm}\frac{\partial^2 p_{jkm}(\mathbf{Q})}{\partial q_{jkm}^2} < 0 \tag{20}$$

Take $\frac{\partial^2 \pi_{icm}(\mathbf{Q})}{\partial q_{icm}^2}$ as an example (We can apply a similar derivation to the other two partial derivatives to show SOC always hold for them as well). As $1+s_{cm} > 0$, sign of $\frac{\partial^2 \pi_{icm}(\mathbf{Q})}{\partial q_{icm}^2}$ is the same as the sign of $2\frac{\partial p_{icm}(\mathbf{Q})}{\partial q_{icm}} + q_{icm}\frac{\partial^2 p_{icm}(\mathbf{Q})}{\partial q_{icm}^2}$.

$$\begin{aligned}
&2\frac{\partial p_{icm}(\mathbf{Q})}{\partial q_{icm}} + q_{icm}\frac{\partial^2 p_{icm}(\mathbf{Q})}{\partial q_{icm}^2}\\
&= 2\left(-\frac{1}{\sigma}\frac{Y_m q_{icm}^{-\frac{1+\sigma}{\sigma}} A_{icm}^{\frac{\sigma-1}{\sigma}}}{(q_{0mm}A_{0mm})^{\frac{\sigma-1}{\sigma}} + \sum_{c'}\sum_{i'}(q_{i'c'm}A_{i'c'm})^{\frac{\sigma-1}{\sigma}}}\frac{1}{q_{icm}} - \frac{\sigma-1}{\sigma}\frac{Y_m q_{icm}^{-\frac{2}{\sigma}} A_{icm}^{\frac{2\sigma-2}{\sigma}}}{\left((q_{0mm}A_{0mm})^{\frac{\sigma-1}{\sigma}} + \sum_{c'}\sum_{i'}(q_{i'c'm}A_{i'c'm})^{\frac{\sigma-1}{\sigma}}\right)^2}\right)\\
&+ q_{icm}\left(\frac{1+\sigma}{\sigma^2}\frac{Y_m q_{icm}^{-\frac{1+2\sigma}{\sigma}} A_{icm}^{\frac{\sigma-1}{\sigma}}}{(q_{0mm}A_{0mm})^{\frac{\sigma-1}{\sigma}} + \sum_{c'}\sum_{i'}(q_{i'c'm}A_{i'c'm})^{\frac{\sigma-1}{\sigma}}}\frac{1}{q_{icm}} + \frac{\sigma-1}{\sigma^2}\frac{Y_m q_{icm}^{-\frac{2+\sigma}{\sigma}} A_{icm}^{\frac{2\sigma-2}{\sigma}}}{\left((q_{0mm}A_{0mm})^{\frac{\sigma-1}{\sigma}} + \sum_{c'}\sum_{i'}(q_{i'c'm}A_{i'c'm})^{\frac{\sigma-1}{\sigma}}\right)^2}\right)\\
&+ q_{icm}\left(\frac{2\sigma-2}{\sigma^2}\frac{Y_m q_{icm}^{-\frac{2+\sigma}{\sigma}} A_{icm}^{\frac{2\sigma-2}{\sigma}}}{\left((q_{0mm}A_{0mm})^{\frac{\sigma-1}{\sigma}} + \sum_{c'}\sum_{i'}(q_{i'c'm}A_{i'c'm})^{\frac{\sigma-1}{\sigma}}\right)^2} + \frac{2(\sigma-1)^2}{\sigma^2}\frac{Y_m q_{icm}^{-\frac{3}{\sigma}} A_{icm}^{\frac{3\sigma-3}{\sigma}}}{\left((q_{0mm}A_{0mm})^{\frac{\sigma-1}{\sigma}} + \sum_{c'}\sum_{i'}(q_{i'c'm}A_{i'c'm})^{\frac{\sigma-1}{\sigma}}\right)^3}\right)\\
&= \frac{1-\sigma}{\sigma^2}\frac{Y_m q_{icm}^{-\frac{1+\sigma}{\sigma}} A_{icm}^{\frac{\sigma-1}{\sigma}}}{(q_{0mm}A_{0mm})^{\frac{\sigma-1}{\sigma}} + \sum_{c'}\sum_{i'}(q_{i'c'm}A_{i'c'm})^{\frac{\sigma-1}{\sigma}}} + \frac{(3-2\sigma)(\sigma-1)}{\sigma^2}\frac{Y_m q_{icm}^{-\frac{2}{\sigma}} A_{icm}^{\frac{2\sigma-2}{\sigma}}}{\left((q_{0mm}A_{0mm})^{\frac{\sigma-1}{\sigma}} + \sum_{c'}\sum_{i'}(q_{i'c'm}A_{i'c'm})^{\frac{\sigma-1}{\sigma}}\right)^2}\\
&+ \frac{2(\sigma-1)^2}{\sigma^2}\frac{Y_m q_{icm}^{1-\frac{3}{\sigma}} A_{icm}^{\frac{3\sigma-3}{\sigma}}}{\left((q_{0mm}A_{0mm})^{\frac{\sigma-1}{\sigma}} + \sum_{c'}\sum_{i'}(q_{i'c'm}A_{i'c'm})^{\frac{\sigma-1}{\sigma}}\right)^3}\\
&= \frac{Y_m}{q_{icm}^2}\left(\frac{1-\sigma}{\sigma^2}S_{icm} + \frac{(3-2\sigma)(\sigma-1)}{\sigma^2}S_{icm}^2 + \frac{2(\sigma-1)^2}{\sigma^2}S_{icm}^3\right)
\end{aligned} \tag{21}$$

Define $f(S_{icm}) = \frac{1-\sigma}{\sigma^2}S_{icm} + \frac{(3-2\sigma)(\sigma-1)}{\sigma^2}S_{icm}^2 + \frac{2(\sigma-1)^2}{\sigma^2}S_{icm}^3$, then sign$[\frac{\partial^2 \pi_{ic}(\mathbf{Q})}{\partial q_{icm}^2}]$ = sign$[f(S_{icm})]$

$$f'(S_{icm}) = \frac{1-\sigma}{\sigma^2} + \frac{2(3-2\sigma)(\sigma-1)}{\sigma^2}S_{icm} + \frac{3\cdot 2(\sigma-1)^2}{\sigma^2}S_{icm}^2$$

It is easy to show that $f'(S_{icm})$ is a convex function with maximum value achieving at either $S_{icm} = 0$ or $S_{icm} = 1$, given that $S_{icm} \in (0,1)$ and $\sigma > 1$. $f'(0) = \frac{1-\sigma}{\sigma} < 0$ and $f'(1) = \frac{2\sigma^2-3\sigma+1}{\sigma^2} > 0$. Therefore, $f(S_{icm})$ first decreases and then increases on $(0,1)$, with maximum value achieved at either $S_{icm} = 0$ or $S_{icm} = 1$. Since $f(0) = 0$ and $f(1) = \frac{1-\sigma+(3-2\sigma)(\sigma-1)+2(\sigma-1)^2}{\sigma^2} = 0$, we have sign$[\frac{\partial^2 \pi_{ic}(\mathbf{Q})}{\partial q_{icm}^2}]$ = sign$[f(S_{icm})] < 0$.

Strategic substitutes or complementarity

$$
\begin{aligned}
\frac{\partial^2 \pi_{icm}(\mathbf{Q})}{\partial q_{jkm}\partial q_{icm}} &= (1+s_{cm})\frac{\partial p_{icm}(\mathbf{Q})}{\partial q_{jkm}} + (1+s_{cm})q_{icm}\frac{\partial^2 p_{icm}(\mathbf{Q})}{\partial q_{jkm}\partial q_{icm}} \\
&= -(1+s_{cm})\frac{\sigma-1}{\sigma}\frac{Y_m q_{icm}^{-\frac{1}{\sigma}} A_{icm}^{\frac{\sigma-1}{\sigma}} q_{jkm}^{-\frac{1}{\sigma}} A_{jkm}^{\frac{\sigma-1}{\sigma}}}{(q_{0mm}A_{0mm})^{\frac{\sigma-1}{\sigma}} + \sum_{c'}\sum_{i'}(q_{i'c'm}A_{i'c'm})^{\frac{\sigma-1}{\sigma}}} \\
&+ (1+s_{cm})q_{icm}\left(\frac{\sigma-1}{\sigma^2}\frac{Y_m q_{icm}^{-\frac{1+\sigma}{\sigma}} A_{icm}^{\frac{\sigma-1}{\sigma}} q_{jkm}^{-\frac{1}{\sigma}} A_{jkm}^{\frac{\sigma-1}{\sigma}}}{(q_{0mm}A_{0mm})^{\frac{\sigma-1}{\sigma}} + \sum_{c'}\sum_{i'}(q_{i'c'm}A_{i'c'm})^{\frac{\sigma-1}{\sigma}}} + \left(\frac{\sigma-1}{\sigma}\right)^2 \frac{2Y_m q_{icm}^{-\frac{2}{\sigma}} A_{icm}^{\frac{2\sigma-2}{\sigma}} q_{jkm}^{-\frac{1}{\sigma}} A_{jkm}^{\frac{\sigma-1}{\sigma}}}{(q_{0mm}A_{0mm})^{\frac{\sigma-1}{\sigma}} + \left(\sum_{c'}\sum_{i'}(q_{i'c'm}A_{i'c'm})^{\frac{\sigma-1}{\sigma}}\right)^3}\right) \\
&= -(1+s_{cm})\left(\frac{\sigma-1}{\sigma}\right)^2 \frac{Y_m q_{icm}^{\frac{\sigma-1}{\sigma}} A_{icm}^{\frac{\sigma-1}{\sigma}} q_{jkm}^{\frac{\sigma-1}{\sigma}} A_{jkm}^{\frac{\sigma-1}{\sigma}}}{\left((q_{0mm}A_{0mm})^{\frac{\sigma-1}{\sigma}} + \sum_{c'}\sum_{i'}(q_{i'c'm}A_{i'c'm})^{\frac{\sigma-1}{\sigma}}\right)^2}\frac{1}{q_{icm}q_{jkm}}\left(1 - 2\frac{(q_{icm}A_{icm})^{\frac{\sigma-1}{\sigma}}}{(q_{0mm}A_{0mm})^{\frac{\sigma-1}{\sigma}} + \sum_{c'}\sum_{i'}(q_{i'c'm}A_{i'c'm})^{\frac{\sigma-1}{\sigma}}}\right) \\
&= -(1+s_{cm})\left(\frac{\sigma-1}{\sigma}\right)^2 \frac{1}{Y_m} p_{icm} p_{jkm}(1-2S_{icm})
\end{aligned}
\tag{22}
$$

$$\frac{\partial^2 \pi_{icm}(\mathbf{Q})}{\partial q_{jkm}\partial q_{icm}} < 0 \text{ if } S_{icm} = \frac{(q_{icm}A_{icm})^{\frac{\sigma-1}{\sigma}}}{\sum_{c'}\sum_{i'}(q_{i'c'm}A_{i'c'm})^{\frac{\sigma-1}{\sigma}}} < 0.5.$$

Total differentiation of FOC

$$\frac{\partial^2 \pi_{icm}(\mathbf{Q})}{\partial q_{icm}^2}\,\mathrm{d}q_{icm} + \sum_{i'\in\mathcal{I}_c, i'\neq i}\frac{\partial^2 \pi_{icm}(\mathbf{Q})}{\partial q_{i'cm}\partial q_{icm}}\,\mathrm{d}q_{i'cm} + \sum_{c'\neq c}\sum_{i'\in\mathcal{I}_{c'}}\frac{\partial^2 \pi_{icm}(\mathbf{Q})}{\partial q_{i'c'm}\partial q_{icm}}\,\mathrm{d}q_{i'c'm} + \frac{\partial^2 \pi_{icm}(\mathbf{Q})}{\partial q_{0mm}\partial q_{icm}}\,\mathrm{d}q_{0mm} + \frac{\partial^2 \pi_{icm}(\mathbf{Q})}{\partial s_{cm}\partial q_{icm}}\,\mathrm{d}s_{cm} = 0, \quad \forall m \neq c$$

$$\frac{\partial^2 \pi_{icc}(\mathbf{Q})}{\partial q_{icc}^2}\,\mathrm{d}q_{icc} + \sum_{i'\in\mathcal{I}_c, i'\neq i}\frac{\partial^2 \pi_{icc}(\mathbf{Q})}{\partial q_{i'cc}\partial q_{icc}}\,\mathrm{d}q_{i'cc} + \sum_{c'\neq c}\sum_{i'\in\mathcal{I}_{c'}}\frac{\partial^2 \pi_{icc}(\mathbf{Q})}{\partial q_{i'c'c}\partial q_{icc}}\,\mathrm{d}q_{i'c'c} + \frac{\partial^2 \pi_{icc}(\mathbf{Q})}{\partial q_{0cc}\partial q_{icc}}\,\mathrm{d}q_{0cc} = 0$$

$$\frac{\partial^2 \pi_{jkm}(\mathbf{Q})}{\partial q_{jkm}^2}\,\mathrm{d}q_{jkm} + \sum_{i'\in\mathcal{I}_c, i'\neq i}\frac{\partial^2 \pi_{jkm}(\mathbf{Q})}{\partial q_{i'cm}\partial q_{jkm}}\,\mathrm{d}q_{i'cm} + \sum_{c'\neq c}\sum_{i'\in\mathcal{I}_{c'}}\frac{\partial^2 \pi_{jkm}(\mathbf{Q})}{\partial q_{i'c'm}\partial q_{jkm}}\,\mathrm{d}q_{i'c'm} + \frac{\partial^2 \pi_{jkm}(\mathbf{Q})}{\partial q_{0mm}\partial q_{jkm}}\,\mathrm{d}q_{0mm} + \frac{\partial^2 \pi_{jkm}(\mathbf{Q})}{\partial s_{cm}\partial q_{jkm}}\,\mathrm{d}s_{cm} = 0, \quad \forall m, k \neq c$$

For a given market $m \neq c$, there are $N_m^* = 1 + \sum_{c=1}^{C} N_c$ equations. In matrix form, the system of equations is

$$
\begin{bmatrix}
\frac{\partial^2 \pi_{11m}(\mathbf{Q})}{\partial q_{11m}^2} & \cdots & \frac{\partial^2 \pi_{11m}(\mathbf{Q})}{\partial q_{N_C Cm}\partial q_{11m}} & \frac{\partial^2 \pi_{11m}(\mathbf{Q})}{\partial q_{0mm}\partial q_{11m}} \\
\vdots & \ddots & \vdots & \vdots \\
\frac{\partial^2 \pi_{1cm}(\mathbf{Q})}{\partial q_{11m}\partial q_{1cm}} & \cdots & \frac{\partial^2 \pi_{1cm}(\mathbf{Q})}{\partial q_{N_C Cm}\partial q_{1cm}} & \frac{\partial^2 \pi_{1cm}(\mathbf{Q})}{\partial q_{0mm}\partial q_{1cm}} \\
\vdots & \ddots & \vdots & \vdots \\
\frac{\partial^2 \pi_{N_c cm}(\mathbf{Q})}{\partial q_{11m}\partial q_{N_c cm}} & \cdots & \frac{\partial^2 \pi_{N_c cm}(\mathbf{Q})}{\partial q_{N_C Cm}\partial q_{N_c cm}} & \frac{\partial^2 \pi_{N_c cm}(\mathbf{Q})}{\partial q_{0mm}\partial q_{N_c cm}} \\
\vdots & \ddots & \vdots & \vdots \\
\frac{\partial^2 \pi_{N_C Cm}(\mathbf{Q})}{\partial q_{11m}\partial q_{N_C Cm}} & \cdots & \frac{\partial^2 \pi_{N_C Cm}(\mathbf{Q})}{\partial q_{N_C Cm}^2} & \frac{\partial^2 \pi_{N_C Cm}(\mathbf{Q})}{\partial q_{0mm}\partial q_{N_C Cm}} \\
\frac{\partial^2 \pi_{0mm}(\mathbf{Q})}{\partial q_{11m}\partial q_{0mm}} & \cdots & \frac{\partial^2 \pi_{0mm}(\mathbf{Q})}{\partial q_{N_C Cm}\partial q_{0mm}} & \frac{\partial^2 \pi_{0mm}(\mathbf{Q})}{\partial q_{0mm}^2}
\end{bmatrix}
\begin{bmatrix}
\mathrm{d}q_{11m}/\,\mathrm{d}s_{cm} \\ \vdots \\ \mathrm{d}q_{1cm}/\,\mathrm{d}s_{cm} \\ \vdots \\ \mathrm{d}q_{N_c cm}/\,\mathrm{d}s_{cm} \\ \vdots \\ \mathrm{d}q_{N_C Cm}/\,\mathrm{d}s_{cm} \\ \mathrm{d}q_{0mm}/\,\mathrm{d}s_{cm}
\end{bmatrix}
=
\begin{bmatrix}
-\frac{\partial^2 \pi_{11m}(\mathbf{Q})}{\partial s_{cm}\partial q_{11m}} \\ \vdots \\ -\frac{\partial^2 \pi_{1cm}(\mathbf{Q})}{\partial s_{cm}\partial q_{1cm}} \\ \vdots \\ -\frac{\partial^2 \pi_{N_c cm}(\mathbf{Q})}{\partial s_{cm}\partial q_{N_c cm}} \\ \vdots \\ -\frac{\partial^2 \pi_{N_C Cm}(\mathbf{Q})}{\partial s_{cm}\partial q_{N_C Cm}} \\ -\frac{\partial^2 \pi_{0mm}(\mathbf{Q})}{\partial s_{cm}\partial q_{0mm}}
\end{bmatrix}
=
\begin{bmatrix}
0 \\ \vdots \\ -\frac{\sigma-1}{\sigma} p_{icm}(1-S_{icm}) \\ \vdots \\ -\frac{\sigma-1}{\sigma} p_{icm}(1-S_{icm}) \\ \vdots \\ 0 \\ 0
\end{bmatrix}
\tag{23}
$$

Denote $M = \begin{bmatrix} \frac{\partial^2 \pi_{11m}(\mathbf{Q})}{\partial q_{11m}^2} & \cdots & \frac{\partial^2 \pi_{11m}(\mathbf{Q})}{\partial q_{N_C Cm} \partial q_{11m}} & \frac{\partial^2 \pi_{11m}(\mathbf{Q})}{\partial q_{0mm} \partial q_{11m}} \\ \vdots & \ddots & \vdots & \vdots \\ \frac{\partial^2 \pi_{1cm}(\mathbf{Q})}{\partial q_{11m} \partial q_{1cm}} & \cdots & \frac{\partial^2 \pi_{1cm}(\mathbf{Q})}{\partial q_{N_C Cm} \partial q_{1cm}} & \frac{\partial^2 \pi_{1cm}(\mathbf{Q})}{\partial q_{0mm} \partial q_{1cm}} \\ \vdots & \ddots & \vdots & \vdots \\ \frac{\partial^2 \pi_{N_c cm}(\mathbf{Q})}{\partial q_{11m} \partial q_{N_c cm}} & \cdots & \frac{\partial^2 \pi_{N_c cm}(\mathbf{Q})}{\partial q_{N_C Cm} \partial q_{N_c cm}} & \frac{\partial^2 \pi_{N_c cm}(\mathbf{Q})}{\partial q_{0mm} \partial q_{N_c cm}} \\ \vdots & \ddots & \vdots & \vdots \\ \frac{\partial^2 \pi_{N_C Cm}(\mathbf{Q})}{\partial q_{11m} \partial q_{N_C Cm}} & \cdots & \frac{\partial^2 \pi_{N_C Cm}(\mathbf{Q})}{\partial q_{N_C Cm}^2} & \frac{\partial^2 \pi_{N_C Cm}(\mathbf{Q})}{\partial q_{0mm} \partial q_{N_C Cm}} \\ \frac{\partial^2 \pi_{0mm}(\mathbf{Q})}{\partial q_{11m} \partial q_{0mm}} & \cdots & \frac{\partial^2 \pi_{0mm}(\mathbf{Q})}{\partial q_{N_C Cm} \partial q_{0mm}} & \frac{\partial^2 \pi_{0mm}(\mathbf{Q})}{\partial q_{0mm}^2} \end{bmatrix}$. Adopting the Cramer's rule,

$$\mathrm{d}q_{jkm} / \, \mathrm{d}s_{cm} = \frac{\det(M_{jk})}{\det(M)}$$

where M_{jk} is the matrix M with the $[j + \sum_{c=1}^{k-1} N_c]$th column replaced by the column vector $\begin{bmatrix} 0 \\ \vdots \\ -\frac{\sigma-1}{\sigma} p_{1cm}(1 - S_{1cm}) \\ \vdots \\ -\frac{\sigma-1}{\sigma} p_{N_c cm}(1 - S_{N_c cm}) \\ \vdots \\ 0 \\ 0 \end{bmatrix}$.

Applying the determinant formula and dropping the zero terms,[10]

$$\begin{aligned} \det(M_{jk}) &= -\sum_{i=1}^{N_c} (-1)^{i + \sum_{l=1}^{c-1} N_l + j + \sum_{n=1}^{k-1} N_n} \frac{\partial^2 \pi_{icm}(\vec{Q})}{\partial s_{cm} \partial q_{icm}} \det(M^*_{jk,ic}) \\ &= \sum_{i=1}^{N_c} (-1)^{i + \sum_{l=1}^{c-1} N_l + j + \sum_{n=1}^{k-1} N_n} \frac{\sigma - 1}{\sigma} p_{icm}(1 - S_{icm}) \det(M^*_{jk,ic}) \end{aligned}$$

where $M^*_{jk,ic}$ is the matrix M_{jk} with the $[i + \sum_{l=1}^{c-1} N_l]$th row and $[j + \sum_{n=1}^{k-1} N_n]$th column removed.

[10] $M = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & \dots & a_{ij} & \dots & a_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nj} & \dots & a_{nn} \end{pmatrix}$, then $\det(M) = \sum_{i=1}^{n} (-1)^{i+j} a_{ij} \det(M_{ij})$, where M_{ij} is the matrix M with the ith row and jth column removed.

Similarly,

$$\mathrm{d}q_{0mm}/\,\mathrm{d}s_{cm} = \frac{\det(M_{N_m^* N_m^*})}{\det(M)}$$

where $M_{N_m^* N_m^*}$ is the matrix M with the last column replaced by the column vector $\begin{bmatrix} 0 \\ \vdots \\ -1 \\ \vdots \\ -1 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$.

$$\det(M_{N_m^* N_m^*}) = \sum_{i=1}^{N_c} (-1)^{i+\sum_{l=1}^{c-1} N_l + 1 + \sum_{n=1}^{C} N_n} \det(M^*_{N_m^* N_m^*, ic})$$

where $M^*_{N_m^* N_m^*, ic}$ is the matrix $M_{N_m^* N_m^*}$ with the $[i+\sum_{l=1}^{c-1} N_l]$th row and $[N_m^*]$th (last) column removed.

A.1.2 First stage (given $\frac{dq_{jkm}}{ds_{cm}}$ for $\forall k, m, j \in \mathcal{I}_k$, and $\frac{dq_{0mm}}{ds_{cm}}$):

Focus on the welfare of country c gained from market $m \neq c$,

$$W_c(s_{cm}) = \sum_{i \in \mathcal{I}_c} \pi_{icm}(\mathbf{Q}) - s_{cm} \sum_{i \in \mathcal{I}_c} p_{icm}(\mathbf{Q}) q_{icm}$$

$$\begin{aligned}
\frac{\mathrm{d}W_c(s_{cm})}{\mathrm{d}s_{cm}} &= \sum_{i \in \mathcal{I}_c} \frac{\mathrm{d}\pi_{icm}(\mathbf{Q})}{\mathrm{d}s_{cm}} - \sum_{i \in \mathcal{I}_c} p_{icm}(\mathbf{Q}) q_{icm} - s_{cm} \sum_{i \in \mathcal{I}_c} \left(q_{icm} \frac{\mathrm{d}p_{icm}}{\mathrm{d}s_{cm}} + p_{icm} \frac{\mathrm{d}q_{icm}}{\mathrm{d}s_{cm}} \right) \\
&= \sum_{i \in \mathcal{I}_c} \frac{\partial \pi_{icm}(\mathbf{Q})}{\partial q_{icm}} \frac{\mathrm{d}q_{icm}}{\mathrm{d}s_{cm}} + \sum_{i \in \mathcal{I}_c} \sum_{k \neq c} \sum_{j \in \mathcal{I}_k} \frac{\partial \pi_{icm}(\mathbf{Q})}{\partial q_{jkm}} \frac{\mathrm{d}q_{jkm}}{\mathrm{d}s_{cm}} \\
&+ \sum_{i \in \mathcal{I}_c} \sum_{j \neq i} \frac{\partial \pi_{icm}(\mathbf{Q})}{\partial q_{jcm}} \frac{\mathrm{d}q_{jcm}}{\mathrm{d}s_{cm}} + \sum_{i \in \mathcal{I}_c} \frac{\partial \pi_{icm}(\mathbf{Q})}{\partial q_{0mm}} \frac{\mathrm{d}q_{0mm}}{\mathrm{d}s_{cm}} \\
&- s_{cm} \sum_{i \in \mathcal{I}_c} \left(p_{icm} \frac{\mathrm{d}q_{icm}}{\mathrm{d}s_{cm}} + q_{icm} \sum_{k} \sum_{j \in \mathcal{I}_k} \left(\frac{\partial p_{icm}}{\partial q_{jkm}} \frac{\mathrm{d}q_{jkm}}{\mathrm{d}s_{cm}} + \frac{\partial p_{icm}}{\partial q_{0mm}} \frac{\mathrm{d}q_{0mm}}{\mathrm{d}s_{cm}} \right) \right) = 0
\end{aligned} \tag{24}$$

Optimal s^C_{cm} is solved as

$$s^C_{cm} = \frac{\sum_{i\in\mathcal{I}_c} \frac{\partial \pi_{icm}(\mathbf{Q})}{\partial q_{0mm}} \frac{\mathrm{d}q_{0mm}}{\mathrm{d}s_{cm}} + \sum_{i\in\mathcal{I}_c} \sum_{j\neq i} \frac{\partial \pi_{icm}(\mathbf{Q})}{\partial q_{jcm}} \frac{\mathrm{d}q_{jcm}}{\mathrm{d}s_{cm}} + \sum_{i\in\mathcal{I}_c} \sum_{k\neq c} \sum_{j\in\mathcal{I}_k} \frac{\partial \pi_{icm}(\mathbf{Q})}{\partial q_{jkm}} \frac{\mathrm{d}q_{jkm}}{\mathrm{d}s_{cm}}}{\sum_{i\in\mathcal{I}_c} \left(p_{icm} \frac{\mathrm{d}q_{icm}}{\mathrm{d}s_{cm}} + q_{icm} \sum_k \sum_{j\in\mathcal{I}_k} \left(\frac{\partial p_{icm}}{\partial q_{jkm}} \frac{\mathrm{d}q_{jkm}}{\mathrm{d}s_{cm}} + \frac{\partial p_{icm}}{\partial q_{0mm}} \frac{\mathrm{d}q_{0mm}}{\mathrm{d}s_{cm}} \right) \right)} \tag{25}$$

A.2 Bertrand

The revenue of firm i from country c in market m is

$$R_{icm}(\mathbf{P}) = p_{icm} q_{icm} = Y_m \frac{p_{icm}^{1-\sigma} A_{icm}^{\sigma-1}}{\left(\frac{p_{0mm}}{A_{0mm}}\right)^{1-\sigma} + \sum_{c'} \sum_{i'} \left(\frac{p_{i'c'm}}{A_{i'c'm}}\right)^{1-\sigma}}$$

The profit of firm i from country c with export subsidy is

$$\begin{aligned}
\pi_{ic}(\mathbf{P}) &= \sum_m \left[R_{icm}(\mathbf{P}) - c_{icm} q_{icm}\right] + \sum_{m\neq c} s_{cm} p_{icm} q_{icm} \\
&= \sum_m \left[Y_m \frac{p_{icm}^{1-\sigma} A_{icm}^{\sigma-1}}{\left(\frac{p_{0mm}}{A_{0mm}}\right)^{1-\sigma} + \sum_{c'} \sum_{i'} \left(\frac{p_{i'c'm}}{A_{i'c'm}}\right)^{1-\sigma}} - c_{icm} Y_m \frac{p_{icm}^{-\sigma} A_{icm}^{\sigma-1}}{\left(\frac{p_{0mm}}{A_{0mm}}\right)^{1-\sigma} + \sum_{c'} \sum_{i'} \left(\frac{p_{i'c'm}}{A_{i'c'm}}\right)^{1-\sigma}} \right] \\
&+ \sum_{m\neq c} \left[s_{cm} Y_m \frac{p_{icm}^{1-\sigma} A_{icm}^{\sigma-1}}{\left(\frac{p_{0mm}}{A_{0mm}}\right)^{1-\sigma} + \sum_{c'} \sum_{i'} \left(\frac{p_{i'c'm}}{A_{i'c'm}}\right)^{1-\sigma}} - c_{icm} Y_m \frac{p_{icm}^{-\sigma} A_{icm}^{\sigma-1}}{\left(\frac{p_{0mm}}{A_{0mm}}\right)^{1-\sigma} + \sum_{c'} \sum_{i'} \left(\frac{p_{i'c'm}}{A_{i'c'm}}\right)^{1-\sigma}} \right]
\end{aligned}$$

The profit of firm j from another country k is

$$\begin{aligned}
\pi_{jk}(\mathbf{P}) &= \sum_m \left[R_{jkm}(\mathbf{P}) - c_{jkm} q_{jkm}\right] \\
&= \sum_m \left[Y_m \frac{p_{jkm}^{1-\sigma} A_{jkm}^{\sigma-1}}{\left(\frac{p_{0mm}}{A_{0mm}}\right)^{1-\sigma} + \sum_{c'} \sum_{i'} \left(\frac{p_{i'c'm}}{A_{i'c'm}}\right)^{1-\sigma}} - c_{jkm} Y_m \frac{p_{jkm}^{-\sigma} A_{jkm}^{\sigma-1}}{\left(\frac{p_{0mm}}{A_{0mm}}\right)^{1-\sigma} + \sum_{c'} \sum_{i'} \left(\frac{p_{i'c'm}}{A_{i'c'm}}\right)^{1-\sigma}} \right]
\end{aligned}$$

The welfare of country c is

$$W_c(s) = \sum_{i\in\mathcal{I}_c} \sum_m \pi_{icm}(\mathbf{P}) - \sum_{i\in\mathcal{I}_c} \sum_{m\neq c} s_{cm} Y_m \frac{p_{icm}^{-\sigma} A_{icm}^{\sigma-1}}{\left(\frac{p_{0mm}}{A_{0mm}}\right)^{1-\sigma} + \sum_{c'} \sum_{i'} \left(\frac{p_{i'c'm}}{A_{i'c'm}}\right)^{1-\sigma}}$$

The own price elasticity of demand is

$$\epsilon^B_{icm} \equiv -\left(\frac{\partial q_{icm}}{\partial p_{icm}}\frac{p_{icm}}{q_{icm}}\right) = -\left(\sigma Y_m \frac{p_{icm}^{-\sigma-1}A_{icm}^{\sigma-1}}{\left(\frac{p_{0mm}}{A_{0mm}}\right)^{1-\sigma} + \sum_{c'}\sum_{i'\in\Omega_{c'm}}\left(\frac{p_{i'c'm}}{A_{i'c'm}}\right)^{1-\sigma}}\right.$$
$$\left. + (1-\sigma)Y_m \frac{p_{icm}^{-2\sigma}A_{icm}^{2\sigma-2}}{\left(\left(\frac{p_{0mm}}{A_{0mm}}\right)^{1-\sigma} + \sum_{c'}\sum_{i'\in\Omega_{c'm}}\left(\frac{p_{i'c'm}}{A_{i'c'm}}\right)^{1-\sigma}\right)^2}\right)\frac{p_{icm}}{q_{icm}}$$
$$= \sigma + (1-\sigma)S_{icm}(\mathbf{p}_m, \boldsymbol{\xi}_m)$$

A.2.1 Second stage (given s_{cm}):

First-order conditions

The first-order condition of firm i from country c in market $m \neq c$ is

$$\frac{\partial \pi_{icm}(\mathbf{P})}{\partial p_{icm}} = (1+s_{cm})q_{icm} + (1+s_{cm})p_{icm}\frac{\partial q_{icm}(\mathbf{P})}{\partial p_{icm}} - c_{icm}\frac{\partial q_{icm}(\mathbf{P})}{\partial p_{icm}}$$
$$= (1+s_{cm})Y_m \frac{p_{icm}^{-\sigma}A_{icm}^{\sigma-1}}{\left(\frac{p_{0mm}}{A_{0mm}}\right)^{1-\sigma} + \sum_{c'}\sum_{i'}\left(\frac{p_{i'c'm}}{A_{i'c'm}}\right)^{1-\sigma}}$$
$$- ((1+s_{cm})p_{icm} - c_{icm})\left(\sigma Y_m \frac{p_{icm}^{-\sigma-1}A_{icm}^{\sigma-1}}{\left(\frac{p_{0mm}}{A_{0mm}}\right)^{1-\sigma} + \sum_{c'}\sum_{i'}\left(\frac{p_{i'c'm}}{A_{i'c'm}}\right)^{1-\sigma}} + (1-\sigma)Y_m \frac{p_{icm}^{-2\sigma}A_{icm}^{2\sigma-2}}{\left(\left(\frac{p_{0mm}}{A_{0mm}}\right)^{1-\sigma} + \sum_{c'}\sum_{i'}\left(\frac{p_{i'c'm}}{A_{i'c'm}}\right)^{1-\sigma}\right)^2}\right)$$
$$= (1+s_{cm})Y_m \frac{p_{icm}^{-\sigma}A_{icm}^{\sigma-1}}{\left(\frac{p_{0mm}}{A_{0mm}}\right)^{1-\sigma} + \sum_{c'}\sum_{i'}\left(\frac{p_{i'c'm}}{A_{i'c'm}}\right)^{1-\sigma}}\left(1 - \frac{(1+s_{cm})p_{icm} - c_{icm}}{(1+s_{cm})p_{icm}}\left(\sigma + (1-\sigma)\frac{p_{icm}^{1-\sigma}A_{icm}^{\sigma-1}}{\left(\frac{p_{0mm}}{A_{0mm}}\right)^{1-\sigma} + \sum_{c'}\sum_{i'}\left(\frac{p_{i'c'm}}{A_{i'c'm}}\right)^{1-\sigma}}\right)\right)$$
$$= (1+s_{cm})Y_m S_{icm}\frac{1}{p_{icm}}\left(1 - \frac{(1+s_{cm})p_{icm} - c_{icm}}{(1+s_{cm})p_{icm}}(\sigma + (1-\sigma)S_{icm})\right) = 0 \tag{26}$$

Similarly, the first-order condition of firm i from country c in market c is

$$\frac{\partial \pi_{icc}(\mathbf{Q})}{\partial p_{icc}} = Y_m S_{icc}\frac{1}{p_{icc}}\left(1 - \frac{p_{icc} - c_{icc}}{p_{icc}}(\sigma + (1-\sigma)S_{icc})\right) = 0 \tag{27}$$

The first-order condition of firm j from another country $k \neq c$ in any market m (including local firms in market m) is

$$\frac{\partial \pi_{jkm}(\mathbf{Q})}{\partial p_{jkm}} = Y_m S_{jkm}\frac{1}{p_{jkm}}\left(1 - \frac{p_{jkm} - c_{jkm}}{p_{jkm}}(\sigma + (1-\sigma)S_{jkm})\right) = 0 \tag{28}$$

Second-order conditions

$$\frac{\partial^2 \pi_{icm}(\mathbf{P})}{\partial p_{icm}^2} = 2(1+s_{cm})\frac{\partial q_{icm}(\mathbf{P})}{\partial p_{icm}} + (1+s_{cm})p_{icm}\frac{\partial^2 q_{icm}(\mathbf{P})}{\partial p_{icm}^2} - c_{icm}\frac{\partial^2 q_{icm}(\mathbf{P})}{\partial p_{icm}^2} < 0 \tag{29}$$

$$\frac{\partial^2 \pi_{icc}(\mathbf{P})}{\partial p_{icc}^2} = 2\frac{\partial q_{icc}(\mathbf{P})}{\partial p_{icc}} + p_{icc}\frac{\partial^2 q_{icc}(\mathbf{P})}{\partial p_{icc}^2} - c_{icc}\frac{\partial^2 q_{icc}(\mathbf{P})}{\partial p_{icc}^2} < 0 \tag{30}$$

$$\frac{\partial^2 \pi_{jkm}(\mathbf{P})}{\partial p_{jkm}^2} = 2\frac{\partial q_{jkm}(\mathbf{P})}{\partial p_{jkm}} + p_{jkm}\frac{\partial^2 q_{jkm}(\mathbf{P})}{\partial p_{jkm}^2} - c_{jkm}\frac{\partial^2 q_{jkm}(\mathbf{P})}{\partial p_{jkm}^2} < 0 \tag{31}$$

Take $\frac{\partial^2 \pi_{icm}(\mathbf{Q})}{\partial p_{icm}^2}$ as an example.

$$\begin{aligned}\frac{\partial^2 \pi_{icm}(\mathbf{P})}{\partial p_{icm}^2} &= 2(1+s_{cm})\frac{\partial q_{icm}(\mathbf{P})}{\partial p_{icm}} + (1+s_{cm})p_{icm}\frac{\partial^2 q_{icm}(\mathbf{P})}{\partial p_{icm}^2} - c_{icm}\frac{\partial^2 q_{icm}(\mathbf{P})}{\partial p_{icm}^2} \\ &= -2(1+s_{cm})Y_m S_{icm}\frac{1}{p_{icm}^2}\left(\sigma + (1-\sigma)S_{icm}\right) \\ &+ ((1+s_{cm})p_{icm} - c_{icm})Y_m\frac{1}{p_{icm}^3}\left(\sigma(1+\sigma)S_{icm} + 3\sigma(1-\sigma)S_{icm}^2 + 2(1-\sigma)^2 S_{icm}^3\right)\end{aligned} \tag{32}$$

$$\begin{aligned}\frac{\partial^2 \pi_{icm}(\mathbf{Q})}{\partial p_{icm}^2} < 0 \quad \text{iff} \quad \frac{2(1+s_{cm})p_{icm}}{(1+s_{cm})p_{icm} - c_{icm}} &> \frac{\sigma(\sigma+1) + 3\sigma(1-\sigma)S_{icm} + 2(\sigma-1)^2 S_{icm}^2}{\sigma + (1-\sigma)S_{icm}} \\ &= 2(1-\sigma)S_{icm} + \sigma + \frac{\sigma}{\sigma + (1-\sigma)S_{icm}}\end{aligned}$$

Plug in FOC, $1 - \frac{(1+s_{cm})p_{icm} - c_{icm}}{(1+s_{cm})p_{icm}}\left(\sigma + (1-\sigma)S_{icm}\right) = 0$, then $\frac{\partial^2 \pi_{icm}(\mathbf{Q})}{\partial p_{icm}^2} < 0$ iff

$$2\sigma + 2(1-\sigma)S_{icm} > 2(1-\sigma)S_{icm} + \sigma + \frac{\sigma}{\sigma + (1-\sigma)S_{icm}}$$

$$\sigma > \frac{\sigma}{\sigma + (1-\sigma)S_{icm}}$$

$$\sigma + (1-\sigma)S_{icm} = (1-S_{icm})\sigma + S_{icm} > 1$$

Since $1 - S_{icm} > 0$, $\sigma > 1$, we have $(1-S_{icm})\sigma + S_{icm} > (1-S_{icm}) \times 1 + S_{icm} = 1$. Therefore, $\frac{\partial^2 \pi_{icm}(\mathbf{Q})}{\partial p_{icm}^2} < 0$ is always satisfied.

Strategic complements (given FOC)

$$\begin{aligned}
\frac{\partial^2 \pi_{icm}(\mathbf{P})}{\partial p_{jkm} \partial p_{icm}} &= (1+s_{cm})\frac{\partial q_{icm}(\mathbf{P})}{\partial p_{jkm}} + (1+s_{cm}) p_{icm} \frac{\partial^2 q_{icm}(\mathbf{P})}{\partial p_{jkm}\partial p_{icm}} - c_{icm}\frac{\partial^2 q_{icm}(\mathbf{P})}{\partial p_{jkm}\partial p_{icm}} \\
&= -(1+s_{cm})(1-\sigma) Y_m S_{icm} S_{jkm} \frac{1}{p_{icm} p_{jkm}} \left(1 - \frac{p_{icm} - c_{icm} + s_{cm}}{p_{icm}} \left(\sigma + (1-\sigma) S_{icm}\right)\right) \\
&+ (1+s_{cm})(1-\sigma)^2 Y_m \frac{p_{icm} - c_{icm} + s_{cm}}{p_{icm}} S_{icm}^2 S_{jkm} \frac{1}{p_{icm} p_{jkm}} \\
&= (1-\sigma)^2 Y_m \frac{p_{icm} - c_{icm} + s_{cm}}{p_{icm}} S_{icm}^2 S_{jkm} \frac{1}{p_{icm} p_{jkm}} > 0
\end{aligned} \tag{33}$$

Total differentiation of FOC

$$\frac{\partial^2 \pi_{ic}(\mathbf{P})}{\partial p_{icm}^2}\,\mathrm{d}p_{icm} + \sum_{i' \in \mathcal{I}_c, i' \neq i} \frac{\partial^2 \pi_{ic}(\mathbf{P})}{\partial p_{i'cm}\partial p_{icm}}\,\mathrm{d}p_{i'cm} + \sum_{c' \neq c}\sum_{i' \in \mathcal{I}_{c'}} \frac{\partial^2 \pi_{ic}(\mathbf{P})}{\partial p_{i'c'm}\partial p_{icm}}\,\mathrm{d}p_{i'c'm} + \frac{\partial^2 \pi_{ic}(\mathbf{P})}{\partial p_{0mm}\partial p_{icm}}\,\mathrm{d}p_{0mm} + \frac{\partial^2 \pi_{ic}(\mathbf{P})}{\partial s_c \partial p_{icm}}\,\mathrm{d}s_{cm} = 0, \quad \forall m \neq c$$

$$\frac{\partial^2 \pi_{ic}(\mathbf{P})}{\partial p_{icc}^2}\,\mathrm{d}p_{icc} + \sum_{i' \in \mathcal{I}_c, i' \neq i} \frac{\partial^2 \pi_{ic}(\mathbf{P})}{\partial p_{i'cc}\partial p_{icc}}\,\mathrm{d}p_{i'cc} + \sum_{c' \neq c}\sum_{i' \in \mathcal{I}_{c'}} \frac{\partial^2 \pi_{ic}(\mathbf{P})}{\partial p_{i'c'c}\partial p_{icc}}\,\mathrm{d}p_{i'c'c} + \frac{\partial^2 \pi_{ic}(\mathbf{P})}{\partial p_{0cc}\partial p_{icc}}\,\mathrm{d}p_{0cc} = 0$$

$$\frac{\partial^2 \pi_{jk}(\mathbf{P})}{\partial p_{jkm}^2}\,\mathrm{d}p_{jkm} + \sum_{i' \in \mathcal{I}_c, i' \neq i} \frac{\partial^2 \pi_{jk}(\mathbf{P})}{\partial p_{i'cm}\partial p_{jkm}}\,\mathrm{d}p_{i'cm} + \sum_{c' \neq c}\sum_{i' \in \mathcal{I}_{c'}} \frac{\partial^2 \pi_{jk}(\mathbf{P})}{\partial p_{i'c'm}\partial p_{jkm}}\,\mathrm{d}p_{i'c'm} + \frac{\partial^2 \pi_{jk}(\mathbf{P})}{\partial p_{0mm}\partial p_{jkm}}\,\mathrm{d}p_{0mm} + \frac{\partial^2 \pi_{jk}(\mathbf{P})}{\partial s_c \partial p_{jkm}}\,\mathrm{d}s_{cm} = 0, \quad \forall m, k \neq c$$

For a given market $m \neq c$, there are $N_m^* = 1 + \sum_{c=1}^{C} N_c$ equations. In matrix form, the system of equations is

$$\begin{bmatrix}
\frac{\partial^2 \pi_{11m}(\mathbf{P})}{\partial p_{11m}^2} & \cdots & \frac{\partial^2 \pi_{11m}(\mathbf{P})}{\partial p_{N_C Cm}\partial p_{11m}} & \frac{\partial^2 \pi_{11m}(\mathbf{P})}{\partial p_{0mm}\partial p_{11m}} \\
\vdots & \ddots & \vdots & \vdots \\
\frac{\partial^2 \pi_{1cm}(\mathbf{P})}{\partial p_{11m}\partial p_{1cm}} & \cdots & \frac{\partial^2 \pi_{1cm}(\mathbf{P})}{\partial p_{N_C Cm}\partial p_{1cm}} & \frac{\partial^2 \pi_{1cm}(\mathbf{P})}{\partial p_{0mm}\partial p_{1cm}} \\
\vdots & \ddots & \vdots & \vdots \\
\frac{\partial^2 \pi_{N_c cm}(\mathbf{P})}{\partial p_{11m}\partial p_{N_c cm}} & \cdots & \frac{\partial^2 \pi_{N_c cm}(\mathbf{P})}{\partial p_{N_C Cm}\partial p_{N_c cm}} & \frac{\partial^2 \pi_{N_c cm}(\mathbf{P})}{\partial p_{0mm}\partial p_{N_c cm}} \\
\vdots & \ddots & \vdots & \vdots \\
\frac{\partial^2 \pi_{N_C Cm}(\mathbf{P})}{\partial p_{11m}\partial p_{N_C Cm}} & \cdots & \frac{\partial^2 \pi_{N_C Cm}(\mathbf{P})}{\partial p_{N_C Cm}^2} & \frac{\partial^2 \pi_{N_C Cm}(\mathbf{P})}{\partial p_{0mm}\partial p_{N_C Cm}} \\
\frac{\partial^2 \pi_{0mm}(\mathbf{P})}{\partial p_{11m}\partial p_{0mm}} & \cdots & \frac{\partial^2 \pi_{0mm}(\mathbf{P})}{\partial p_{N_C Cm}\partial p_{0mm}} & \frac{\partial^2 \pi_{0mm}(\mathbf{P})}{\partial p_{0mm}^2}
\end{bmatrix}
\begin{bmatrix}
\frac{\mathrm{d}p_{11m}}{\mathrm{d}s_{cm}} \\ \vdots \\ \frac{\mathrm{d}p_{1cm}}{\mathrm{d}s_{cm}} \\ \vdots \\ \frac{\mathrm{d}p_{N_c cm}}{\mathrm{d}s_{cm}} \\ \vdots \\ \frac{\mathrm{d}p_{N_C Cm}}{\mathrm{d}s_{cm}} \\ \frac{\mathrm{d}p_{0mm}}{\mathrm{d}s_{cm}}
\end{bmatrix}
=
\begin{bmatrix}
-\frac{\partial^2 \pi_{11m}(\mathbf{Q})}{\partial s_{cm}\partial p_{11m}} \\ \vdots \\ -\frac{\partial^2 \pi_{1cm}(\mathbf{Q})}{\partial s_{cm}\partial p_{1cm}} \\ \vdots \\ -\frac{\partial^2 \pi_{N_c cm}(\mathbf{Q})}{\partial s_{cm}\partial p_{N_c cm}} \\ \vdots \\ -\frac{\partial^2 \pi_{N_C Cm}(\mathbf{Q})}{\partial s_{cm}\partial p_{N_C Cm}} \\ -\frac{\partial^2 \pi_{0mm}(\mathbf{Q})}{\partial s_{cm}\partial p_{0mm}}
\end{bmatrix}
=
\begin{bmatrix}
0 \\ \vdots \\ -Y_m RS_{1cm}\frac{1}{p_{1cm}}(1-\sigma)(1-RS_{1cm}) \\ \vdots \\ -Y_m RS_{N_c cm}\frac{1}{p_{N_c cm}}(1-\sigma)(1-RS_{N_c cm}) \\ \vdots \\ 0 \\ 0
\end{bmatrix} \tag{34}$$

Denote $M = \begin{bmatrix} \frac{\partial^2 \pi_{11m}(\mathbf{P})}{\partial p_{11m}^2} & \cdots & \frac{\partial^2 \pi_{11m}(\mathbf{P})}{\partial p_{N_C Cm} \partial p_{11m}} & \frac{\partial^2 \pi_{11m}(\mathbf{P})}{\partial p_{0mm} \partial p_{11m}} \\ \vdots & \ddots & \vdots & \vdots \\ \frac{\partial^2 \pi_{1cm}(\mathbf{P})}{\partial p_{11m} \partial p_{1cm}} & \cdots & \frac{\partial^2 \pi_{1cm}(\mathbf{P})}{\partial p_{N_C Cm} \partial p_{1cm}} & \frac{\partial^2 \pi_{1cm}(\mathbf{P})}{\partial p_{0mm} \partial p_{1cm}} \\ \vdots & \ddots & \vdots & \vdots \\ \frac{\partial^2 \pi_{N_c cm}(\mathbf{P})}{\partial p_{11m} \partial p_{N_c cm}} & \cdots & \frac{\partial^2 \pi_{N_c cm}(\mathbf{P})}{\partial p_{N_C Cm} \partial p_{N_c cm}} & \frac{\partial^2 \pi_{N_c cm}(\mathbf{P})}{\partial p_{0mm} \partial p_{N_c cm}} \\ \vdots & \ddots & \vdots & \vdots \\ \frac{\partial^2 \pi_{N_C Cm}(\mathbf{P})}{\partial p_{11m} \partial p_{N_C Cm}} & \cdots & \frac{\partial^2 \pi_{N_C Cm}(\mathbf{P})}{\partial p_{N_C Cm}^2} & \frac{\partial^2 \pi_{N_C Cm}(\mathbf{P})}{\partial p_{0mm} \partial p_{N_C Cm}} \\ \frac{\partial^2 \pi_{0mm}(\mathbf{P})}{\partial p_{11m} \partial p_{0mm}} & \cdots & \frac{\partial^2 \pi_{0mm}(\mathbf{P})}{\partial p_{N_C Cm} \partial p_{0mm}} & \frac{\partial^2 \pi_{0mm}(\mathbf{P})}{\partial p_{0mm}^2} \end{bmatrix}$, then the solution is

$$\mathrm{d}p_{jkm} / \, \mathrm{d}s_{cm} = \frac{\det(M_{jk})}{\det(M)}$$

where M_{jk} is the matrix M with the $[j + \sum_{c=1}^{k-1} N_c]$th column replaced by the column vector $\begin{bmatrix} 0 \\ \vdots \\ -Y_m RS_{1cm} \frac{1}{p_{1cm}} (1-\sigma)(1 - RS_{1cm}) \\ \vdots \\ -Y_m RS_{N_c cm} \frac{1}{p_{N_c cm}} (1-\sigma)(1 - RS_{N_c cm}) \\ \\ \vdots \\ 0 \\ 0 \end{bmatrix}$.

Applying the determinant formula and dropping the zero terms,

$$\begin{aligned} \det(M_{jk}) &= -\sum_{i=1}^{N_c} (-1)^{i + \sum_{l=1}^{c-1} N_l + j + \sum_{n=1}^{k-1} N_n} \frac{\partial^2 \pi_{icm}(\mathbf{Q})}{\partial s_{cm} \partial p_{icm}} \det(M^*_{jk,ic}) \\ &= \sum_{i=1}^{N_c} (-1)^{i + \sum_{l=1}^{c-1} N_l + j + \sum_{n=1}^{k-1} N_n} Y_m S_{icm} \frac{1}{p_{icm}} (1-\sigma)(1 - S_{icm}) \det(M^*_{jk,ic}) \end{aligned}$$

where $M^*_{jk,ic}$ is the matrix M_{jk} with the $[i + \sum_{l=1}^{c-1} N_l]$th row and $[j + \sum_{n=1}^{k-1} N_n]$th column removed.

Similarly,

$$\mathrm{d}p_{0mm} / \, \mathrm{d}s_{cm} = \frac{\det(M_{N^*_m N^*_m})}{\det(M)}$$

$$\begin{aligned} \det(M_{N^*_m N^*_m}) &= \sum_{i=1}^{N_c} (-1)^{i + \sum_{l=1}^{c-1} N_l + 1 + \sum_{n=1}^{C} N_n} \frac{\partial^2 \pi_{icm}(\mathbf{Q})}{\partial s_{cm} \partial p_{icm}} \det(M^*_{N^*_m N^*_m, ic}) \\ &= \sum_{i=1}^{N_c} (-1)^{i + \sum_{l=1}^{c-1} N_l + 1 + \sum_{n=1}^{C} N_n} Y_m S_{icm} \frac{1}{p_{icm}} (1-\sigma)(1 - S_{icm}) \det(M^*_{N^*_m N^*_m, ic}) \end{aligned}$$

where $M^*_{N^*_m N^*_m, ic}$ is the matrix $M_{N^*_m N^*_m}$ with the $[i + \sum_{l=1}^{c-1} N_l]$th row and $[N^*_m]$th (last) column removed.

A.2.2 First stage (given $\frac{dp_{jkm}}{ds_{cm}}$ for $\forall k, m, j \in \mathcal{I}_k$, and $\frac{dp_{0mm}}{ds_{cm}}$):

$$\begin{aligned}
\frac{\mathrm{d}W_c(s_{cm})}{\mathrm{d}s_c} &= \sum_{i\in\mathcal{I}_c} \frac{\mathrm{d}\pi_{icm}(\mathbf{P})}{\mathrm{d}s_c} - \sum_{i\in\mathcal{I}_c} p_{icm} q_{icm}(\mathbf{P}) - s_{cm} \sum_{i\in\mathcal{I}_c} \left(p_{icm} \frac{\mathrm{d}q_{icm}}{\mathrm{d}s_{cm}} + q_{icm} \frac{\mathrm{d}p_{icm}}{\mathrm{d}s_{cm}} \right) \\
&= \sum_{i\in\mathcal{I}_c} \frac{\partial\pi_{icm}(\mathbf{P})}{\partial p_{icm}} \frac{\mathrm{d}p_{icm}}{\mathrm{d}s_{cm}} + \sum_{i\in\mathcal{I}_c} \sum_{k\neq c} \sum_{j\in\mathcal{I}_k} \frac{\partial\pi_{icm}(\mathbf{P})}{\partial p_{jkm}} \frac{\mathrm{d}p_{jkm}}{\mathrm{d}s_{cm}} + \sum_{i\in\mathcal{I}_c} \sum_{j\neq c} \frac{\partial\pi_{icm}(\mathbf{P})}{\partial p_{jcm}} \frac{\mathrm{d}p_{jcm}}{\mathrm{d}s_{cm}} \\
&+ \sum_{i\in\mathcal{I}_c} \frac{\partial\pi_{icm}(\mathbf{P})}{\partial p_{0mm}} \frac{\mathrm{d}p_{0mm}}{\mathrm{d}s_{cm}} - s_{cm} \sum_{i\in\mathcal{I}_c} \left(q_{icm} \frac{\mathrm{d}p_{icm}}{\mathrm{d}s_{cm}} + p_{icm} \sum_{k} \sum_{j\in\mathcal{I}_k} \left(\frac{\partial q_{icm}}{\partial p_{jkm}} \frac{\mathrm{d}p_{jkm}}{\mathrm{d}s_{cm}} + \frac{\partial q_{icm}}{\partial p_{0mm}} \frac{\mathrm{d}p_{0mm}}{\mathrm{d}s_{cm}} \right) \right) \\
&= 0
\end{aligned} \tag{35}$$

Optimal s^B_{cm} is solved as

$$s^B_{cm} = \frac{\sum_{i\in\mathcal{I}_c} \frac{\partial\pi_{icm}(\mathbf{P})}{\partial p_{0mm}} \frac{\mathrm{d}p_{0mm}}{\mathrm{d}s_{cm}} + \sum_{i\in\mathcal{I}_c} \sum_{j\neq c} \frac{\partial\pi_{icm}(\mathbf{P})}{\partial p_{jcm}} \frac{\mathrm{d}p_{jcm}}{\mathrm{d}s_{cm}} + \sum_{i\in\mathcal{I}_c} \sum_{k\neq c} \sum_{j\in\mathcal{I}_k} \frac{\partial\pi_{icm}(\mathbf{P})}{\partial p_{jkm}} \frac{\mathrm{d}p_{jkm}}{\mathrm{d}s_{cm}}}{\sum_{i\in\mathcal{I}_c} \left(q_{icm} \frac{\mathrm{d}p_{icm}}{\mathrm{d}s_{cm}} + p_{icm} \sum_k \sum_{j\in\mathcal{I}_k} \left(\frac{\partial q_{icm}}{\partial p_{jkm}} \frac{\mathrm{d}p_{jkm}}{\mathrm{d}s_{cm}} + \frac{\partial q_{icm}}{\partial p_{0mm}} \frac{\mathrm{d}p_{0mm}}{\mathrm{d}s_{cm}} \right) \right)} \tag{36}$$

B Derivation of optimal export subsidy/tax in two-firm third-market model

Continuing from Appendix A, this section discusses optimal export subsidy/tax in the two-firm third-market model.

B.1 Cournot Duopoly

Equation (23) is simplified as

$$\begin{bmatrix} \pi_{xx} & \pi_{xy} \\ \pi^*_{yx} & \pi^*_{yy} \end{bmatrix} \begin{bmatrix} \mathrm{d}x/\,\mathrm{d}s \\ \mathrm{d}y/\,\mathrm{d}s \end{bmatrix} = \begin{bmatrix} -\pi_{xs} \\ -\pi^*_{ys} \end{bmatrix} = \begin{bmatrix} -\frac{\sigma-1}{\sigma} p(1-R_x) \\ 0 \end{bmatrix} \tag{37}$$

Applying Cramer's rule,

$$\frac{\mathrm{d}x}{\mathrm{d}s} = -\frac{\sigma-1}{\sigma}p(1-R_x)\frac{\pi^*_{yy}}{\pi_{xx}\pi^*_{yy}-\pi_{xy}\pi^*_{yx}} \tag{38}$$

$$\frac{\mathrm{d}y}{\mathrm{d}s} = \frac{\sigma-1}{\sigma}p(1-R_x)\frac{\pi^*_{yx}}{\pi_{xx}\pi^*_{yy}-\pi_{xy}\pi^*_{yx}} \tag{39}$$

In the first stage,

$$\frac{\mathrm{d}W}{\mathrm{d}s} = \pi_x\frac{\mathrm{d}x}{\mathrm{d}s}+\pi_y\frac{\mathrm{d}y}{\mathrm{d}s}+\pi_s-x-sp\frac{\mathrm{d}x}{\mathrm{d}s}-sxp_x\frac{\mathrm{d}x}{\mathrm{d}s}-sxp_y\frac{\mathrm{d}y}{\mathrm{d}s} = \pi_y\frac{\mathrm{d}y}{\mathrm{d}s}-sp\frac{\mathrm{d}x}{\mathrm{d}s}-sxp_x\frac{\mathrm{d}x}{\mathrm{d}s}-sxp_y\frac{\mathrm{d}y}{\mathrm{d}s} = 0$$

Optimal s^C is solved as

$$s^C = -\pi_y\frac{\pi^*_{yx}}{p\pi^*_{yy}+xp_x\pi^*_{yy}-xp_y\pi^*_{yx}} \tag{40}$$

$$\begin{aligned}
p\pi^*_{yy}+xp_x\pi^*_{yy}-xp_y\pi^*_{yx} \Leftrightarrow\ & \frac{MR_x}{x}\frac{M}{y^2}\left(\frac{1-\sigma}{\sigma^2}R_y+\frac{(3-2\sigma)(\sigma-1)}{\sigma^2}R_y^2+\frac{2(\sigma-1)^2}{\sigma^2}R_y^3\right)\\
& -x\left(\frac{M}{\sigma x^2}R_x+\frac{\sigma-1}{\sigma}\frac{M}{x^2}R_x^2\right)\left(\frac{1-\sigma}{\sigma^2}R_y+\frac{(3-2\sigma)(\sigma-1)}{\sigma^2}R_y^2+\frac{2(\sigma-1)^2}{\sigma^2}R_y^3\right)\\
& -\frac{\sigma-1}{\sigma}R_xp^*\left(\frac{\sigma-1}{\sigma}\right)^2\frac{1}{M}pp^*\left(1-2R_y\right)\\
& \Leftrightarrow \frac{(\sigma-1)(1-R_y)+1}{\sigma(1-R_y)}\frac{1}{R_y^2}\left(\frac{1-\sigma}{\sigma^2}R_y+\frac{(3-2\sigma)(\sigma-1)}{\sigma^2}R_y^2+\frac{2(\sigma-1)^2}{\sigma^2}R_y^3\right)\\
& -\left(\frac{\sigma-1}{\sigma}\right)^3(1-R_y)(1-2R_y)\\
& \equiv h(R_y)
\end{aligned}$$

Easy to show $h(R_y)<0$ for all $R_y\in(0,1)$ and $\sigma>1$. Thus $p\pi^*_{yy}+xp_x\pi^*_{yy}-xp_y\pi^*_{yx}<0$. Given that $\pi_y<0$ and $p\pi^*_{yy}+xp_x\pi^*_{yy}-xp_y\pi^*_{yx}<0$, we have

$$\text{sign}[s^C] = -\text{sign}[\pi^*_{yx}]$$

$\pi^*_{yx}<0$ iff $R_y<0.5$. Therefore, $s^C>0$ iff $R_y<0.5$.

B.2 Bertrand Duopoly

Equation (34) is simplified as

$$\begin{bmatrix}\pi_{pp} & \pi_{pp^*}\\ \pi^*_{p^*p} & \pi^*_{p^*p^*}\end{bmatrix}\begin{bmatrix}\mathrm{d}p/\,\mathrm{d}s\\ \mathrm{d}p^*/\,\mathrm{d}s\end{bmatrix} = \begin{bmatrix}-\pi_{ps}\\ -\pi^*_{p^*s}\end{bmatrix} = \begin{bmatrix}x(\sigma-1)(1-R_x)\\ 0\end{bmatrix} \tag{41}$$

Applying Cramer's rule,

$$\frac{\mathrm{d}p}{\mathrm{d}s} = \frac{x(\sigma-1)(1-R_x)\pi^*_{p^*p^*}}{\pi_{pp}\pi^*_{p^*p^*} - \pi_{pp^*}\pi^*_{p^*p}} \tag{42}$$

$$\frac{\mathrm{d}p^*}{\mathrm{d}s} = -\frac{x(\sigma-1)(1-R_x)\pi^*_{p^*p}}{\pi_{pp}\pi^*_{p^*p^*} - \pi_{pp^*}\pi^*_{p^*p}} \tag{43}$$

In the first stage, Optimal s^B is solved as

$$\begin{aligned} s^B &= \frac{\pi_{p^*}\frac{\mathrm{d}p^*}{\mathrm{d}s}}{px_p\frac{\mathrm{d}p}{\mathrm{d}s} + px_{p^*}\frac{\mathrm{d}p^*}{\mathrm{d}s} + x\frac{\mathrm{d}p}{\mathrm{d}s}} \\ &= -\frac{\pi_{p^*}\pi^*_{p^*p}}{px_p\pi^*_{yy} + x\pi^*_{yy} - px_{p^*}\pi^*_{yx}} \end{aligned}$$

$$\begin{aligned} px_p\pi^*_{p^*p^*} + x\pi^*_{p^*p^*} - px_{p^*}\pi^*_{p^*p} \Leftrightarrow & -MR_x\frac{1}{p}(\sigma+(1-\sigma)R_x)\left(-2MR_y\frac{1}{(p^*)^2}(\sigma+(1-\sigma)R_y) + (p^*-c^*)M\frac{1}{(p^*)^3}\left(\sigma(\sigma+1)R_y + 3\sigma(1-\sigma)R_y^2 + 2(1-\sigma)^2R_y^3\right)\right) \\ & - (\sigma-1)MR_xR_y\frac{1}{p^*}(1-\sigma)^2M\frac{p^*-c^*}{p^*}R_y^2R_x\frac{1}{pp^*} \\ & - 2MR_xMR_y\frac{1}{p(p^*)^2}(\sigma+(1-\sigma)R_y) + (p^*-c^*)M^2R_x\frac{1}{p(p^*)^3}\left(\sigma(\sigma+1)R_y + 3\sigma(1-\sigma)R_y^2 + 2(1-\sigma)^2R_y^3\right) \\ \Leftrightarrow & (\sigma+(1-\sigma)R_x)\left(2(\sigma+(1-\sigma)R_y) - (p^*-c^*)\frac{1}{p^*}\left(\sigma(\sigma+1) + 3\sigma(1-\sigma)R_y + 2(1-\sigma)^2R_y^2\right)\right) - (\sigma-1)^3\frac{p^*-c^*}{p^*}R_y^2R_x \\ & - 2(\sigma+(1-\sigma)R_y) + (p^*-c^*)\frac{1}{p^*}\left(\sigma(\sigma+1) + 3\sigma(1-\sigma)R_y + 2(1-\sigma)^2R_y^2\right) \\ \Leftrightarrow & (\sigma+(1-\sigma)R_x-1)\left(2(\sigma+(1-\sigma)R_y) - \frac{1}{\sigma+(1-\sigma)R_y}\left(\sigma(\sigma+1) + 3\sigma(1-\sigma)R_y + 2(1-\sigma)^2R_y^2\right)\right) \\ & - (\sigma-1)^3\frac{1}{\sigma+(1-\sigma)R_y}R_y^2R_x \\ \Leftrightarrow & (\sigma+(1-\sigma)R_x-1)\left(2(\sigma+(1-\sigma)(1-R_x)) - \frac{1}{\sigma+(1-\sigma)(1-R_x)}\left(\sigma(\sigma+1) + 3\sigma(1-\sigma)(1-R_x) + 2(1-\sigma)^2(1-R_x)^2\right)\right) \\ & - (\sigma-1)^3\frac{1}{\sigma+(1-\sigma)(1-R_x)}(1-R_x)^2R_x \end{aligned}$$

Therefore,

$$\begin{aligned} &\mathrm{sign}\left[px_p\pi^*_{p^*p^*} + x\pi^*_{p^*p^*} - px_{p^*}\pi^*_{p^*p}\right] \\ &= \mathrm{sign}\left[(\sigma+(1-\sigma)R_x-1)\left(2(\sigma+(1-\sigma)R_y) - \frac{1}{\sigma+(1-\sigma)R_y}\left(\sigma(\sigma+1) + 3\sigma(1-\sigma)R_y + 2(1-\sigma)^2R_y^2\right)\right) - (\sigma-1)^3\frac{1}{\sigma+(1-\sigma)R_y}R_y^2R_x\right] \end{aligned}$$

Define

$$\begin{aligned} f(\sigma) &= (\sigma+(1-\sigma)R_x-1)\left(2(\sigma+(1-\sigma)R_y) - \frac{1}{\sigma+(1-\sigma)R_y}\left(\sigma(\sigma+1) + 3\sigma(1-\sigma)R_y + 2(1-\sigma)^2R_y^2\right)\right) - (\sigma-1)^3\frac{1}{\sigma+(1-\sigma)R_y}R_y^2R_x \\ &= \frac{\sigma(\sigma-1)R_x(\sigma+(1-\sigma)R_x-1) - (\sigma-1)^3R_y^2R_x}{\sigma+(1-\sigma)R_y} \end{aligned}$$

Therefore,

$$\begin{aligned}
\text{sign}\left[px_p\pi^*_{p^*p^*} + x\pi^*_{p^*p^*} - px_{p^*}\pi^*_{p^*p}\right] &= \text{sign}\left[\sigma(\sigma-1)^2(1-R_x) - (\sigma-1)^3 R_y^2\right] \\
&= \text{sign}\left[\sigma(1-R_x) - (\sigma-1)(1-R_x)^2\right] \\
&= \text{sign}\left[\sigma - (\sigma-1)(1-R_x)\right] \\
&= \text{sign}\left[\sigma - (\sigma-1)(1-R_x) - 1\right] \\
&> \text{sign}\left[(\sigma-1)R_x\right] \\
&> 0
\end{aligned}$$

Given that $\pi_{p^*} = ((1+s)p - c)x_{p^*} > 0$, $\pi^*_{p^*p} > 0$, and $px_p\pi^*_{p^*p^*} + x\pi^*_{p^*p^*} - px_{p^*}\pi^*_{p^*p} > 0$, we have

$$\text{sign}[s^B] < 0$$

C A general testing procedure

While in this paper we focus on a very simple CES environment to make Monte Carlos straightforward to implement, in this appendix we show that this particular demand structure is not needed to implement our conduct test. In particular, we briefly sketch out in this section how the Hausman test be adapted to more flexible specifications of demand, marginal cost, and conduct. The key requirement is that the demand model be invertible in the sense of Berry and Haile (2014). More precisely, suppose the demand is given by the following:[11]

$$q_{icm} = N_m \sigma_{ic}(\delta_{icm}, \boldsymbol{\delta}_{-(i,c),m} : \boldsymbol{\gamma})$$

Here, N_m is the size of market m and $\sigma(,;\boldsymbol{\gamma})$ is an unknown function that characterizes the demand for firm (i,c). This demand system depends on J arguments; the first argument is the firm (i,c)'s linear demand index $\delta_{icm} \equiv \boldsymbol{\beta}\mathbf{X}_{icm} - p_{icm} + \xi_{icm}$, where $\mathbf{X}_{icm}$ is vector of observable product characteristics, p_{icm} is the price charged, and ξ_{icm} is a demand shock that is known to consumers and firms, but not the econometrician or policy maker.[12] The remaining $J-1$ arguments are a vector of the demand indexes for all other

[11]We work with a special case of Berry and Haile (2014) primarily to develop intuition; specifically, we assume prices and product characteristics only affect demand through the linear index. This can be relaxed, but at the cost of further notation.

[12]The fact that price enters with a coefficient of 1 is a normalization, since we can choose the units of ξ_{icm} so that this holds while representing the same set of preferences; see Berry and Haile (2014).

firms $\boldsymbol{\delta}_{-(i,c)m} \equiv \{\delta_{jkm}\}_{(j,k)\neq(i,c)}$.[13] Berry and Haile (2014) show that under fairly weak conditions, demand systems of this form are invertible, in the sense that there exists an inverse function $\sigma_{ic}^{-1}(:, \boldsymbol{\gamma})$ such that:

$$\delta_{icm} = \sigma_{ic}^{-1}\left(\mathbf{S}_m; \boldsymbol{\gamma}\right) \tag{44}$$

where $\mathbf{S}_m$ is a $J \times 1$ vector of market shares $\frac{q_{icm}}{N_m}$. This can be used a basis for generating the following demand estimation equation:

$$p_{icm} = \boldsymbol{\beta}\mathbf{X}_{icm} - \sigma_{ic}^{-1}\left(\mathbf{S}_m; \boldsymbol{\gamma}\right) + \xi_{icm} \tag{45}$$

Berry and Haile (2014) show that the above model can be identified using a series of supply side instruments that shift costs, τ_{icm}, as well as a instruments based on the observable product characteristics of *other* firms, X_{-icm}.[14] This generates a vector of demand estimates, $\widehat{\boldsymbol{\theta}}^D = \left(\widehat{\boldsymbol{\beta}}^D, \widehat{\boldsymbol{\gamma}}^D\right)$, which can immediately be used to the recover an estimate of the demand shocks $\widehat{\xi}_{icm} = p_{icm} - \widehat{\boldsymbol{\beta}}^D\mathbf{X}_{icm} + \sigma_{ic}^{-1}\left(\mathbf{S}_m; \widehat{\boldsymbol{\gamma}}^D\right)$.

The supply side of the model for single product firms revolves around the specification of a marginal cost function C_{icm} and conduct model R through a markup function $\mu_{icm}^R\left(\mathbf{S}_m; \boldsymbol{\gamma}\right)$.[15] Suppose that marginal costs take the following form:

$$C_{icm} = g(\boldsymbol{\tau}_{icm}; \boldsymbol{\delta})\exp(\omega_{icm})$$

where $g(:, \boldsymbol{\delta})$ is an unknown function parameterized by unknown vector $\boldsymbol{\delta}$, and $\boldsymbol{\tau}_{icm}$ is a vector of supply side variables (tariffs, distance, shipping prices, etc...), that we assume are uncorrelated with productivity shocks ω_{icm}. We can the generate an estimating equation for both supply and demand side parameters by imposing the conduct model:

[13]Note that usually these models also require the existence of an outside option good whose demand index is normalized to 0 (or some other number). In our third market model developed earlier, this role was played by the domestic firms in each market m. We leave the outside option implicit in the specification of $\sigma_{ic}(.)$.

[14]These other firm observable product characteristic instruments are excluded from firm i's demand index. Berry and Haile (2014) show that this class of instrumental variable is crucial for identifying the inverse market share function $\sigma_{ic}^{-1}\left(\mathbf{S}_m; \gamma\right)$. In general, validity of these instruments requires that product characteristics be chosen *before* demand shocks are realized and that demand shocks not be intrinsically correlated with product characteristics.

[15]Note that which markups are generally functions of the prices, product characteristics, and demand shocks of all competitors in a market, since we restrict attention to models with the demand inversion property (44), then it immediately follows that the effect of prices, product characteristics, and demand shocks on demand and markups will only occur through market shares, $\mathbf{S}_m$.

$$p_{icm} = \mu^R_{icm}\left(\mathbf{S}_m;\boldsymbol{\gamma}\right) g(\boldsymbol{\tau}_{icm};\boldsymbol{\delta}) \exp(\omega_{icm})$$

Or after taking logs:

$$\ln p_{icm} = \ln \mu^R_{icm}\left(\mathbf{S}_m;\boldsymbol{\gamma}\right) + \ln g(\boldsymbol{\tau}_{icm};\boldsymbol{\delta}) + \omega_{icm} \tag{46}$$

Here, the supply side can be identified by applying a standard nonlinear GMM estimator to the above model.[16] Estimating Equation (45) and Equation (46) simultaneous, as we proposed for the simpler model above, however, can potentially generate more efficient estimates conditional on imposing the correct pricing model R. Let $\widehat{\boldsymbol{\theta}}^R = \left(\widehat{\boldsymbol{\beta}}^R, \widehat{\boldsymbol{\gamma}}^R\right)$ denote the demand-side parameters recovered from this procedure. The Hausman test for a particular conduct model R is then given by

$$H = \left(\widehat{\boldsymbol{\theta}}^D - \widehat{\boldsymbol{\theta}}^R\right)' \left(\text{Var}\left(\widehat{\boldsymbol{\theta}}^D - \widehat{\boldsymbol{\theta}}^R\right)\right)^{-1} \left(\widehat{\boldsymbol{\theta}}^D - \widehat{\boldsymbol{\theta}}^R\right).$$

Hausman (1978) shows the test statistic H asymptotically has a chi-square distribution with degrees of freedom equal to the number of elements of $\left(\widehat{\boldsymbol{\beta}}, \widehat{\boldsymbol{\gamma}}\right)$.

D Simulation setup

We simulate C production origins and each has N exporting firms. There are additional M markets without any exporters. There is a local firm in each of the $C+M$ markets that produces and sells goods locally.

The demand functions are parameterized as

$$q_{icm} = Y_m \frac{p_{icm}^{-\sigma} \exp\left(\xi_{icm}\right)}{p_{0mm}^{1-\sigma} \exp\left(\xi_{0mm}\right) + \sum_{c'} \sum_{j \in \Omega_{c'm}} p_{jc'm}^{1-\sigma} \exp\left(\xi_{jc'm}\right)}$$

$$q_{0mm} = Y_m \frac{1}{p_{0mm}^{1-\sigma} \exp\left(\xi_{0mm}\right) + \sum_{c'} \sum_{j \in \Omega_{c'm}} p_{jc'm}^{1-\sigma} \exp\left(\xi_{jc'm}\right)}$$

[16] Note, however, $\boldsymbol{\tau}_{icm}$ alone is not sufficient to identify the model. In particular, since exogeneity of $\boldsymbol{\tau}_{icm}$ is needed to pin down the unknown marginal cost function $g(.)$, if we wish to also identify $\mu^R(\mathbf{S}_m;\boldsymbol{\gamma})$, further instruments are needed. Note, however, that we can solve this problem by noting that exogenous of product characteristics $\mathbf{X}_m$ as well as cost shifters for *other* firms $\boldsymbol{\tau}_{-(i,c),m}$, can be used to shift the endogenous S_{icm}. Together $\boldsymbol{\tau}_{icm}$, $\boldsymbol{\tau}_{-(i,c),m}$, and the vector of observed product characteristics $\mathbf{X}_m$ can be used as instruments to identify $\mu^R(\mathbf{S}_m;\boldsymbol{\gamma})$ and $g(\boldsymbol{\tau}_{icm};\boldsymbol{\delta})$.

The marginal costs are parameterized as

$$C_{icm} = \tau_{c(i)m} \exp\left(\omega_{icm}\right)$$

$$C_{0mm} = \tau_{mm} \exp\left(\omega_{0mm}\right) = \exp\left(\omega_{0mm}\right)$$

In simulations, we set $\sigma = 4$. The exogenous variables for each market m are simulated as follows.

- $\ln\left(Y_m\right) \sim N\left(\mu^Y, V^Y\right)$: market size
- $\xi_{icm} \sim N\left(\mu^\xi, V^\xi\right)$: demand shifter of importing goods
- $\omega_{icm} \sim N\left(\mu^\omega, V^\omega\right)$: cost shifter of exporters
- $\xi_{0mm} \sim N\left(\mu^{\xi_0}, V^{\xi_0}\right)$: demand shifter of local goods
- $\omega_{0mm} \sim N\left(\mu^\omega, V^\omega\right)$: cost shifter of local firms
- $\tau_{cm} = 1 + \exp(\psi)$ for $c \neq m$ with $\psi \sim N\left(\psi^\omega, V^\psi\right)$ and $\tau_{mm} = 1$: bilateral trade costs

All $C + M$ markets are independent and the distribution parameters are common across markets in one simulation. We set different distribution parameters of ξ for goods from local firms and importing goods, which allows us to adjust the average market share of local firms.

Equilibrium prices in each market are solved by the fixed-point iteration method. Specifically,

$$\mathbf{p}_m = \boldsymbol{\mu}_m^R\left(\mathbf{p}_m, \boldsymbol{\xi}_m\right) \circ \mathbf{MC}_m$$

where $\mathbf{p}_m$ is a vector of prices, $\boldsymbol{\xi}_m$ is a vector of demand shifters, $\mathbf{MC}_m$ is a vector of realized marginal costs, and $\boldsymbol{\mu}_m^R$ is the markup rule given a conduct. Both the left-hand side and the right-hand side are vectors of the same dimension. To solve for the equilibrium prices, we first guess that equilibrium prices solve the monopolistic competition pricing rule $\mathbf{p}_m^0 = \frac{\sigma}{\sigma-1}\mathbf{MC}_m$. Then, for $g = 0, 1, 2, \ldots$,

1. Calculate $\mathbf{P}_m^N = \boldsymbol{\mu}_m^R\left(\mathbf{P}_m^g, \boldsymbol{\xi}_m\right) \circ \mathbf{MC}_m$
2. Check if $\mathbf{P}_m^N = \mathbf{P}_m^g$. If yes, stop.
3. If not, generate $\mathbf{P}_m^{g+1} = w\mathbf{P}_m^g + (1-w)\mathbf{P}_m^N$, and return to step 1. w is the damping factor.

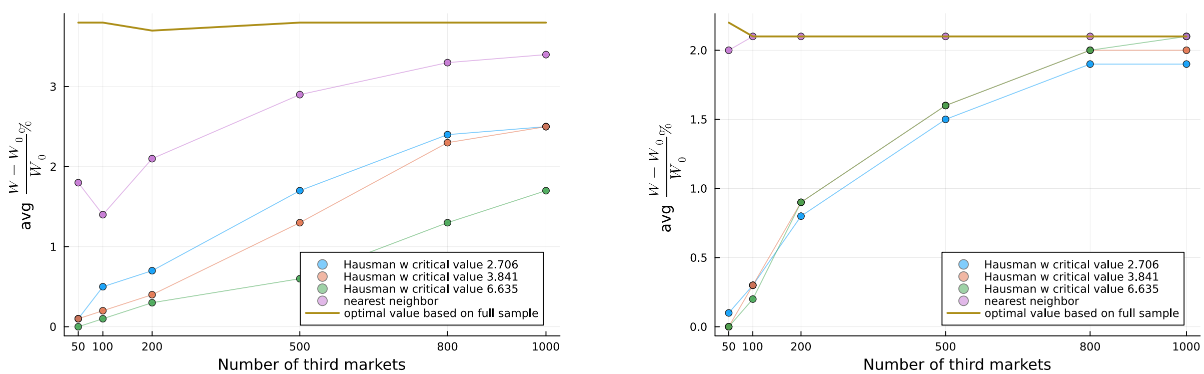
The equilibrium quantities are then calculated by plugging equilibrium prices into demand functions.

For the classical two-firm third-market model, we have a slightly different setting. We set $C = 2$, $N = 1$, and $M = 1$, and simulate the data for the third market only without any local firms. The equilibrium prices are still solved by the fixed-point iteration method.

E Generalized case

We extend the welfare analysis to $C = 3$ production origins and $N = 1$ exporting firms for each origin. Figure 8 shows the average welfare change $\frac{W-W_0}{W_0}\%$ for both Bertrand and Cournot DGPs. The exogenous parameters are the same as $C = 2$, $N = 1$ simulation case, shown in Table 1.

Figure 8: Average welfare change $\frac{W-W_0}{W_0}\%$, generalized cases, $C = 3$, $N = 1$



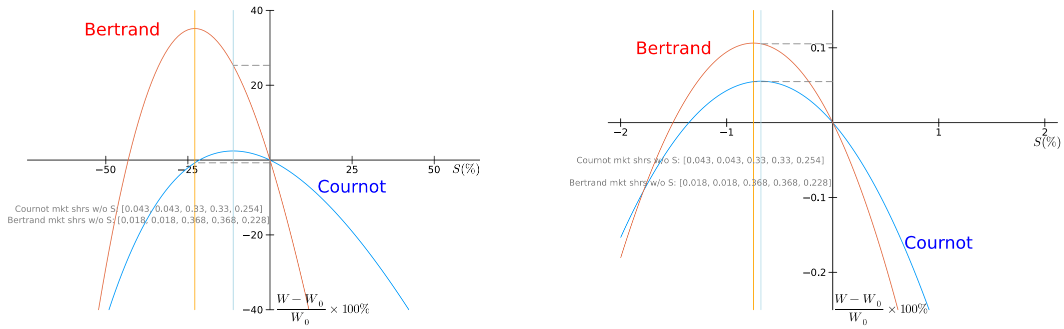
(a) Bertrand DGP

(b) Cournot DGP

Note: This figure visualizes the average welfare change under different numbers of third-markets and inference approaches, for both Bertrand and Cournot DGPs. The Hausman test is conducted with 1%, 5%, and 10% significance levels. The nearest neighbor approach selects the model whose estimate $\hat{\sigma}^R$ is closest to the demand IV estimate $\hat{\sigma}^D$. The average welfare change is calculated as the average of $\frac{W-W_0}{W_0}\%$ over all cases in each simulation. If the policy-maker cannot infer the conduct, the optimal export subsidy or tax is set to zero, and the welfare change is zero.

F Additional figures

Figure 9: C = 2, N = 2, with local firm

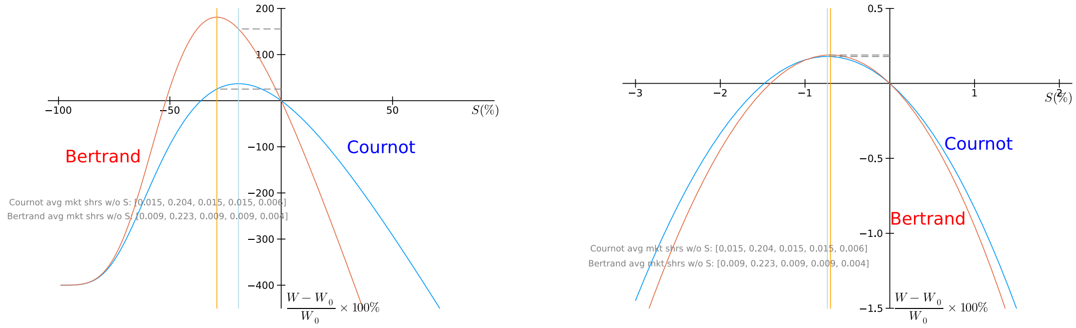


(a) Larger exporting country imposes export tax

(b) Smaller exporting country imposes export tax

Note: This figure visualizes welfare changes of a country that imposes different export subsidies or taxes in terms of price in the extended third-market model, given conduct and a set of exogenous parameters. The rival country and the "third-market" do not impose any export subsidy or tax. Denote the two production origins as country 1 and country 2, and the "third-market" as country 3. We set $\sigma = 5$, $\exp(\xi_{113}) = 1.0$, $\exp(\xi_{213}) = 1.0$, $\exp(\xi_{123}) = 2.0$, $\exp(\xi_{223}) = 2.0$, $\exp(\xi_{033}) = 1.0$, $\exp(\omega_{113}) = \exp(\omega_{213}) = \exp(\omega_{123}) = \exp(\omega_{223}) = 1.0$, $\tau_{13} = 10.0$, $\tau_{23} = 5.0$, $\tau_{33} = 1.0$, $Y_3 = 10.0$. Equilibrium prices, quantities and optimal export subsidies or taxes are solved via fixed-point iteration.

Figure 10: C = 4, N = 4, with local firm



(a) Larger exporting country imposes export tax

(b) Smaller exporting country imposes export tax

Note: This figure visualizes welfare changes of a country that imposes different export subsidies or taxes in terms of price in the extended third-market model, given conduct and a set of exogenous parameters. The rival countries and the "third-market" do not impose any export subsidy or tax. Denote the four production origins as country 1, 2, 3, and 4, and the "third-market" as country 5. We set $\sigma = 5$, $\exp(\xi_{ic5}) = 1.0$ for $i = 1, 2, 3, 4$ and $c = 1, 3, 4$, $\exp(\xi_{i25}) = 2.0$ for $i = 1, 2, 3, 4$, $\exp(\xi_{055}) = 1.0$, $\exp(\omega_{ic5}) = 1.0$ for $i = 1, 2, 3, 4$ and $c = 1, 2, 3, 4$, $\exp(\omega_{055}) = 5.0$, $\tau_{c5} = 4.0$ for $c = 1, 3, 4$, $\tau_{25} = 2.0$, $\tau_{55} = 1.0$, $Y_5 = 10.0$. (a) assumes that country 2 imposes optimal trade policy, and (b) assumes that country 1 imposes optimal trade policy. Equilibrium prices, quantities and optimal export subsidies or taxes are solved via fixed-point iteration.